

Algebra Type - 1

if $ax + \frac{b}{x} = k$ $ax - \frac{b}{x} = \pm \sqrt{k^2 - 4ab}$	$x + \frac{1}{x} = k$ $x - \frac{1}{x} = \pm \sqrt{k^2 - 4}$
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$ax - \frac{b}{x} = k$ $ax + \frac{b}{x} = \pm \sqrt{k^2 + 4ab}$	$x - \frac{1}{x} = k$ $x + \frac{1}{x} = \pm \sqrt{k^2 + 4}$
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① If $x + \frac{1}{x} = \frac{17}{4}$, find $x - \frac{1}{x}$

$$\pm \sqrt{\left(\frac{17}{4}\right)^2 - 4}$$

$$\pm \sqrt{\frac{289}{16} - 4}$$

$$\pm \sqrt{\frac{289 - 64}{16}} = \pm \sqrt{\frac{225}{16}}$$

$$\pm \frac{15}{4} \quad \underline{\text{Ans}}$$

② If $x^{\frac{2007}{2007}} = 13$

find $x^{\frac{2007}{2007}} + \frac{1}{x^{\frac{2007}{2007}}}$

$$\pm \sqrt{(13)^2 + 4}$$

$$\pm \sqrt{173} \quad \underline{\text{Ans}}$$

(3) if $x^5 + \frac{1}{x^3} = 3$ find x^5

$$x^5 + \frac{1}{x^5} = (3)$$

$$x^5 - \frac{1}{x^5} = \pm \sqrt{5}$$

$$\frac{2x^5}{2} = \frac{3 \pm \sqrt{5}}{2}$$

$$x^5 = \frac{3 \pm \sqrt{5}}{2} \quad \underline{\text{Ans}}$$

(4) if $3x - \frac{2}{x} = 5$, find $3x + \frac{2}{x}$

$$\text{if } 3x - \frac{2}{x} = (5)$$

$$3x + \frac{2}{x} = ?$$

$$\text{Sol } \pm \sqrt{k^2 + 4AB}$$

$$\pm \sqrt{25 + 4 \times 3 \times 2}$$

$$\pm = 7 \quad \underline{\text{Ans}}$$

(5) if $5x + 3y = 15$ and $2xy = 6$, then the value of $5x - 3y$ is

$$(5x + 3y)^2 - (5x - 3y)^2 = 4 \times 15 \times y$$

$$225 - (5x - 3y)^2 = 180$$

$$45 = (5x - 3y)^2$$

$3\sqrt{5}$ Ans

⑥ if $3p - 5m = 10$ and $pm = 6$ find $(9p^2 - 25m^2)$

$$(3p + 5m)^2 - ((3p - 5m)^2) = 4 \times 15 pm$$

$$-100 = 36$$

$$(3p + 5m)^2 = 460$$

$$3p + 5m = \pm \sqrt{460}$$

$$(3p)^2 - (5m)^2$$

$$(3p - 5m)(3p + 5m)$$

$$\pm 10 \times \sqrt{460}$$

$3p + 5m = \pm \sqrt{460}$ Ans

⑦ if $5x - \frac{1}{4x} = 6$ ($x > 0$), find $25x^2 - \frac{1}{16x^2} = ?$

Solve

$$\left(5x - \frac{1}{4x}\right) \left(5x + \frac{1}{4x}\right)$$

$$6 \times \left(\pm \sqrt{41}\right)$$

$\pm \sqrt{41}$ Ans

$$\pm \sqrt{k^2 + 4AB}$$

$$\pm \sqrt{36 + 4 \times 5 \times \frac{1}{4}}$$

⑧

$$\frac{x}{\sqrt{xy}} = \frac{\sqrt{x} \times \sqrt{x}}{\sqrt{xy}} = \frac{\sqrt{x}}{\sqrt{y}}$$

⑧ if $\frac{x+y}{\sqrt{xy}} = 10\sqrt{2}$, find

$$\frac{x-y}{\sqrt{xy}}$$

$$\frac{\sqrt{x}}{\sqrt{y}} + \frac{\sqrt{y}}{\sqrt{x}} = 10\sqrt{2}$$

$$\frac{\sqrt{x}}{\sqrt{x}} - \frac{\sqrt{y}}{\sqrt{y}}$$

$$\pm \sqrt{200-4}$$

$$\pm \sqrt{196}$$

$$\pm 14 \text{ Ans}$$

9 if $x + \frac{1}{x} = -14$, $x < -1$, find $x^2 - \frac{1}{x^2} = ?$

Solve

$$\left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) = \pm \sqrt{x^2 - 4}$$

$$-14x - 8\sqrt{3}$$

$$112\sqrt{3} \text{ Ans}$$

$$\pm \sqrt{196-4}$$

$$\pm \sqrt{192}$$

$$\pm \sqrt{64 \times 3}$$

$$\pm 8\sqrt{3}$$

10 if $3x - \frac{1}{4x} = 3$, find $4x + \frac{1}{3x} = ?$

$$x \cdot \frac{4}{3} \left(3x - \frac{1}{4x}\right) = 3x \cdot \frac{4}{3}$$

$$4x - \frac{1}{3x} = 4$$

$$\sqrt{6^2 + 4AB}$$

$$\sqrt{16 + 4 \times 4 \times 1}$$

$$\sqrt{\frac{64}{3}} = \pm \frac{8}{\sqrt{3}}$$

11 If $x+1 = x^2$ and $x > 0$, then $2x^4$ is

$$1 + \frac{1}{x} = x$$

$$x - \frac{1}{x} = 1$$

$$2x + 1 = \pm \sqrt{5}$$

$$x = \frac{\sqrt{5}+1}{2}$$

$$x^2 = \frac{3+\sqrt{5}}{2}$$

$$x^4 = \frac{14+6\sqrt{5}}{4}$$

$$2x^4 = 7+3\sqrt{5}$$

12, 3, 5

Type - 2Symbol Change

(2)

If $x + \frac{1}{x-7} = 0$, Find $x - \frac{1}{x-7}$

$$-(x+7) + \frac{1}{x+7} = 7$$

$$x - \frac{1}{x+7} = ?$$

$$\left[(x+7) - \frac{1}{(x+7)} \right] - 7 = ?$$

$$\pm \sqrt{45} - 7$$

$$\pm 3\sqrt{5} - 7 \text{ Ans}$$

(3)

If $\frac{2x^2}{7x} - \frac{4\sqrt{10}x}{7x} + \frac{7}{7x} = 0$ Find $\frac{2x}{7} - \frac{1}{x} = ?$

$\rightarrow x$

$$\frac{2}{7}x - \frac{4}{7}\sqrt{10} + \frac{1}{x} = 0$$

$$\frac{2}{7}x + \frac{1}{x} = \frac{4}{7}\sqrt{10}$$

$$\pm \sqrt{k^2 - 4AB}$$

$$\pm \sqrt{\frac{160}{49} - 4x}$$

$$\sqrt{\frac{104}{49}}$$

$$= \pm \sqrt{\frac{104}{7}}$$

(5)

If $x + \frac{1}{x} = 7$ & $y - \frac{1}{y} = 4$ find $xy + \frac{1}{xy}$

$$x + \frac{1}{x} = 7$$

$$y + \frac{1}{y} = \sqrt{20}$$

$$x - \frac{1}{x} = \sqrt{45}$$

$$y - \frac{1}{y} = 4$$

$$xy + \cancel{\frac{x}{y}} + \cancel{\frac{y}{x}} + \frac{1}{xy} = 7\sqrt{20} = 14\sqrt{5}$$

$$xy - \cancel{\frac{x}{y}} - \cancel{\frac{y}{x}} + \frac{1}{xy} = 4\sqrt{45} = 12\sqrt{5}$$

$$2\left(xy + \frac{1}{xy}\right) = 26\sqrt{5} = 13\sqrt{5} \text{ Ans}$$

Type - 2

Symbol Change

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$\Rightarrow a^2 + b^2 = (a+b)^2 - 2ab$$

$$\Rightarrow a^2 + b^2 = (a-b)^2 + 2ab$$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$\Rightarrow a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$

$$\Rightarrow a^3 - b^3 = (a-b)^3 + 3ab(a-b)$$

$$\begin{aligned}
 a^3 + b^3 &= (a+b)(a^2 + b^2 - ab) \\
 a^3 - b^3 &= (a-b)(a^2 + b^2 + ab) \\
 &\rightarrow a^3 + b^3 = (a+b)(a^2 + b^2 + ab) \\
 &\rightarrow a^2 + b^2 - ab = \frac{a^3 + b^3}{a+b}
 \end{aligned}$$

5) $p + q = 6$ and $pq = 4$, Find $p^3 + q^3$

$$p^3 + q^3 = (p+q)^3 - 3pq(p+q)$$

$$216 - 72$$

$$= 144 \text{ Ans}$$

6) If $a - b = 2$
 $a^3 - b^3 = 80$
 find ab

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$8 = 80 - 3ab \times 2$$

$$-72 = -6ab$$

$$ab = +12 \text{ Ans}$$

7) $x^3 - y^3 = 416$, and $x - y = 8$ find $(x+y)^2$

$$x^2 + y^2 + xy = \frac{x^3 - y^3}{x - y}$$

$$= \frac{416}{8}$$

9 If $x^6 - 512y^6 = (x^2 + Ay)^2 (x^4 - BX^2y^2 + Cy^4)$,
Find $A+B-C$ $(-8-8-64) = -80$

$$(x^2)^3 - (8y^2)^3 = (x^2 - 8y^2)(x^4 + 64y^4 + 8x^2y^2)$$

-8-8-64
-80 Ans

10 If $x+y = 19$ & $xy = 9$, then find
 $x\sqrt{x} + y\sqrt{y}$?

$$x\sqrt{x} = x^1 \times x^{\frac{1}{2}} = x^{\frac{3}{2}} = (\sqrt{x})^3$$

$$(\sqrt{x})^3 + (\sqrt{y})^3 = (\sqrt{x} + \sqrt{y})(x + y - \sqrt{xy})$$

19+6
 $\sqrt{x} + \sqrt{y} = 5$
 $5(19-3)$
 $5 \times 16 = 80$ Ans

2 If $8(x \times y)^3 - (x - y)^3 = (x + 3y)$

$(Ax^2 + Bxy + Cy^2)$, then find out the

$A - B + C$ Value = ?
 $7 - 6 + 3 = ?$

$$(2x+2y)^3 - (x-y)^3 = (x+3y)(Ax^2 + Bxy + Cy^2)$$

$$(x+3y)(4x^2 + 4y^2 + 8xy + x^2 + y^2 - 2xy + 2x^2 - 2y^2)$$

$$(x+3y)(7x^2 + 3y^2 + 6xy)$$

$A, 7 - 6 + 3 = 4$ Ans

3) If $(8(xxy)^3 - 27(x-y)^3 = 15y - x) = Ax^2 + Bxy + Cy^2$

$$A + B + C = ?$$

$$\left((2x+2y)^3 - (3x-3y)^3 \right) - (5y-x) = Ax^2 + Bxy + Cy^2$$

$$x = 1$$

$$y = 1$$

$$64 - 27 - 5 + 1 = A + B + C$$

$$16 \text{ Ans}$$

4) $(x+y)^3 + (2x-2y)^3 = (3x+Ay)(3x^2+Bxy+Cy^2)$

$$A + B + C = ?$$

$$(3x-y)(x^2+y^2+2xy+4x^2+4y^2-8xy) = (2x^2+2y^2) \quad A+B+C$$

$$(3x-y)(3x^2+7y^2-6xy)$$

$$A + B + C =$$

$$-1 - 6 + 7 = 0 \text{ Ans}$$

9) If $x^6 - 512y^6 = (x^2 + Ay^2)(x^4 - Bx^2y^2 + Cy^4)$

$$A + B - C = ?$$

(13)

If $p = 999$

$$\text{of } \sqrt[3]{p^3 + 3p^2 + 3p + 1} = ?$$

$$\sqrt[3]{(p+1)^3}$$

$$p+1$$

$$999+1 = 1000 \text{ Ans}$$

(14)

If $x+y=1$ and $xy(xy-2)=16$ then
the value of $x^4+y^4 = ?$

$$x^2+y^2+2xy=1$$

$$x^2+y^2=1-2xy$$

$$x^4+y^4+2x^2y^2=1+4x^2y^2-4xy$$

$$x^4+y^4=1+2x^2y^2-4xy$$

$$=1+2(xy(xy-2))$$

$$=1+32=33 \text{ Ans}$$

(16)

If $x = 2 + \sqrt{3}$, then find Value of
 $x^4 - 8x^3 + 16x^2$.

$$x = 2 + \sqrt{3}$$

$$x^2 = 7 + 4\sqrt{3}$$

$$x^4 = 49 + 0$$

$$x-4 = -2 + \sqrt{3}$$

$$(x-4)^2 = 7 - 4\sqrt{3}$$

$$x^2(x^2 - 8x + 16)$$

$$x^2(x-4)^2$$

$$(7+4\sqrt{3})(\sqrt{3}-2)^2$$

$$(7+4\sqrt{3})(7-4\sqrt{3})$$

$$\text{Ans} = 1$$

If $x^2 = 17x + y$, $y^2 = x + 17y$ and $x \neq y$
then find $\sqrt{x^2 + y^2 + 1} = ?$

$$x^2 = 17x + y$$

$$y^2 = x + 17y$$

$$x^2 + y^2 = 18(x + y)$$

$$x^2 - y^2 = 16(x - y)$$

$$x + y = 16$$

$$\sqrt{18 \times 16 + 1}$$

$$= \sqrt{288 + 1}$$

$$= 17 \text{ Ans}$$

(17) If $x(x-4) = 2$, then $x^6 - 88x^3 - 13 = ?$

Cube

$$x^2 - 4x = 2$$

$$x^6 - 64x^3 - \overbrace{3 \times 4x^3(2)} = 8$$

$$x^6 - 88x^3 = 8$$

$$8 - 13$$

$$= -5 \quad \text{Ans}$$

(18) If $8a^3 + 27b^3 = 16$ & $2a + 3b = 4$,
then find the value of
 $16a^4 + 81b^4$?

Solve

$$(2a)^4 + (3b)^4 = ?$$

$$4a^2 + 9b^2 + 12ab = 16$$

$$4a^2 + 9b^2 = 8$$

$$16a^4 + 81b^4 + 2 \times 36a^2b^2 = 64$$

$$64 - 2 \times \frac{8}{9} \times \frac{4}{9}$$

$$= 32 \quad \text{Ans}$$

$$\frac{16^4}{4} = 4 \times (4a^2 + 9b^2 - 6ab)$$

$$4 = 16 - 12ab - 6ab$$

$$\frac{-12}{2} = -\frac{18}{3}ab$$

$$ab = \frac{2}{3}$$

(19) If $5x + 9y = 5$ and $125x^3 + 729y^3 = 120$
then the value of the product of

Solve

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$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$5x + 9y = 5$$

Cube

$$120 + 3 \times 45 \times y(5) = 125$$

$$3 \times 45 \times y \times \cancel{5} = \cancel{5}$$

$$xy = \frac{1}{135} \text{ Ans}$$

(20)

$$4x - 7y = 11$$

$$xy = 8$$

$$16x^2 + 49y^2 = ?$$

Solve

$$16x^2 + 49y^2 - 2 \times 28 \times \overset{x \times 8}{y} = 121$$

$$+ 448$$

$$448$$

$$569$$

Ans

(21)

$$49a^2 + 25b^2 = 30$$

$$ab = 1$$

$$7a + 5b = p?$$

$$30 + 2 \times 35 \times 1 = p^2$$

$$p^2 = 100$$

$$p = 10 \text{ Ans}$$

Solve

(22)

$$\text{If } -x + y = 1$$

$$x^2 + y^2 = 2$$

$$x \times y = ?$$

sq

$$2 + 2xy = 1$$

$$2xy = -1$$

$$xy = -\frac{1}{2}$$

$$x^4 + y^4 + 2x \times \frac{1}{2} = 4$$

$$4 - \frac{1}{2}$$

$$= \frac{7}{2} \text{ Ans}$$

(17)

$$16x^2 + y^2 = \frac{65}{36}$$

$$xy = \frac{1}{18}$$

$$64xy^3 + y^3 = ?$$

Solve

$$(4y)^3 + y^3 = (4x + y) \left(\frac{65}{36} - 4x \times \frac{1}{18} \right)$$

$$(4x + y)^2 = \frac{65}{36} + 2 \times 4x \times \frac{1 \times 2}{18 \times 2}$$

$$\frac{9}{4} \times \left(\frac{65}{36} - \frac{8}{36} \right)$$

$$\frac{81}{36} = 94$$

$$\frac{9}{4} \times \frac{57}{36}$$

$$\frac{57}{16} \text{ Ans}$$

(1)

$$\frac{1}{\sqrt[3]{25} - \sqrt[3]{5+1}} = a\sqrt[3]{25} + b\sqrt[3]{5+1}$$

$$1 \times (5^3 + 1)$$

$$= \frac{1}{6} \sqrt[3]{5} + \frac{1}{6}$$

$$\left(\frac{5^2}{9^2} - \frac{1}{9b} + 1 \right) \left(\frac{5}{a} + 1 \right)$$

$$a + b + c = \frac{1}{3} \text{ Ans}$$

$$\begin{aligned} a &= 0 \\ b &= \frac{1}{6} \\ c &= \frac{1}{6} \end{aligned}$$

Type 3 - Zero से equate करना

$$x^2 + y^2 = 0$$

"दो संकेतों का योग यदि शून्य है"

① If $\underbrace{(a-1)^2}_{a^2} + \underbrace{(b+2)^2}_{b^2} + \underbrace{(c-1)^2}_{c^2} = 0$, the value of $a^2 + b^2 + c^2$ is

Solve

$$\begin{aligned} a &= 1 \\ b &= -2 \\ c &= 1 \end{aligned} \quad \begin{aligned} &1 + 4 + 1 \\ &= 6 \text{ Ans} \end{aligned}$$

$$\begin{aligned} (a+b)^2 &= a^2 + b^2 + 2ab \\ (2a+b)^2 &= (4a^2) + b^2 + 2 \times 2ab \end{aligned}$$

If $a^2 + b^2 + c^2 + 96 = 8(a+b) + 2c$,
 $\downarrow 2(4a+4b-8c)$
 $\sqrt{ab - bc + ca} = ?$

$$\underbrace{a^2 - 2(4a)}_{+16} + \underbrace{b^2 - 2(4b)}_{+16} + \underbrace{c^2 + 2(8c)}_{+64} + 96 = 0$$

$$(a-4)^2 + (b-4)^2 + (c+8)^2 = 0$$

$$\begin{aligned} a &= 4 \\ b &= 4 \\ c &= -8 \end{aligned} \quad \begin{aligned} &16 + 32 - 32 \\ &= 4 \text{ Ans} \end{aligned}$$

- ③ If $a^2 + b^2 + 49c^2 + 18 = 2(b - 28c - a)$
 then the value of $(a+b-7c)$ is

$$a^2 + b^2 + 49c^2 + 18 = 2(b - 28c - a)$$

$$a = -1$$

$$b = 1$$

$$c = \frac{28}{49} = \frac{4}{7}$$

$$1 - 1 - 7 \times \frac{4}{7}$$

$$0 + 4$$

$$= 4 \text{ Ans}$$

- ④ $3x(3x-8) + y(y+1) + 4z^2 - 2z + 16\frac{1}{2} = 0$

Find $6x + y + 2z$

$$3x^2 + y^2 + 4z^2 + 16\frac{1}{2} - 2(12x) - 2(\frac{1}{2}y) + 2z$$

$$x = \frac{12}{9} = \frac{4}{3}$$

$$2(12x - \frac{1}{2}y + z)$$

$$y = \frac{1}{2}$$

$$z = \frac{1}{4}$$

$$6 \times \frac{4}{3} - \frac{1}{2} + 2 \times \frac{1}{4} = 8 \text{ Ans}$$

- ⑤ $(a+b-3)^2 + (b+c-2)^2 + (c+a-5)^2 = 0$,

then the value of $\sqrt{(a+c)^a (c+a)^b}$ is

Solve

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$$\begin{array}{r} 3 \quad 0 \\ a + b = 3 \end{array}$$

$$b + c = 2$$

$$c + a = 5$$

$$\frac{2 \left(\underbrace{a+b}_{3} + \underbrace{c}_{2} \right)}{5} = \frac{10}{5}$$

$$c = 2$$

$$b = 0$$

$$a = 3$$

$$\sqrt{2^3 + 5^0}$$

$$\sqrt{8+1} = 3 \text{ Ans}$$

6

If $10x^2 + y^2 + 6xy + 2x + 1 = 0$,

$$\downarrow \\ 2(3x)y$$

Solve

$$ax^2 + y^2 + 2(3x)y + x^2 + 2x + 1 = 0$$

$$\left(\underbrace{3x+y}_0 \right)^2 + \left(\underbrace{x+1}_0 \right)^2 = 0$$

$$x = -1$$

$$y = 3$$

$$3x + y = ?$$

$$-3 + 12$$

$$= 9 \text{ Ans}$$

7

If $8x^2 + y^2 - 12x - 4xy + 9 = 0$

$\downarrow$

$$\downarrow \\ 2(2x)y$$

Solve

$$4x^2 + y^2 - 2(2x)y + 4x^2 - 12x + 9 = 0$$

$$\left(\underbrace{2x-y}_0 \right)^2 + \left(\underbrace{2x-3}_0 \right)^2 = 0$$

$$14x - 5y = ?$$

$$14 \times \frac{3}{2} - 5 \times 3$$

$$21 - 15 = 6 \text{ Ans}$$

$$x = \frac{3}{2}$$

$$y = 3$$

If $x^2 + (x - 2y - 1)^2 = 4y(x + y)$, then what is the value of $x + 3y$

$$x^2 + 4xy + 4y^2 + (x - 2y - 1)^2 = 0$$

$$(x + 2y)^2 + (x - 2y - 1)^2 = 0$$

Type - 48 Important formula

$$(*) \quad a^3 + b^3 + c^3 - 3abc \begin{cases} \rightarrow (a+b+c)(a^2+b^2+c^2-ab-bc-ca) \\ \rightarrow (a+b+c) \frac{1}{2} ((a-b)^2 + (b-c)^2 + (c-a)^2) \\ \rightarrow (a+b+c)(a+b+c)^2 - 3(ab+bc+ca) \end{cases}$$

$$(*) \quad a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} (a-b)^2 + (b-c)^2 + (c-a)^2$$

$$(*) \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$

$$\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)^2 = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + 2\left(\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}\right)$$

$$(*) \quad a^3 + b^3 + c^3 = (a+b+c)^3 - 3(a+b)(b+c)(c+a)$$

$$(1) \quad \text{If } x+y+z = 19, x^2+y^2+z^2 = 133$$

$$xz = y^2$$

$$z-x = ?$$

Solve

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy+yz+zx)$$

$$361 = 133 + 2y(19)$$

$$y = 6$$

$$x^2 + z^2 = 13$$

$$xz = 36 \text{ Ans}$$

$$(d+e+f)^2 = d^2 + e^2 + f^2 + 2(de+ef+fd)?$$

$$Q \quad 196 = 96 + 2Q$$

$$Q = 50 \quad \underline{\text{Ans}}$$

(3) If $(a+b+c)=13$, and $(ab+bc+ca)=54$
find the value of $(a^2+b^2+c^2)$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$

$$169 = Q + 108$$

$$Q = 61 \quad \underline{\text{Ans}}$$

(4) If $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 10$

$$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = 40$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = ?$$

$$\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)^2 = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + 2\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$$

$$100 = Q + 80$$

$$Q = 20 \quad \underline{\text{Ans}}$$

If $a - \frac{1}{a} = b$, $b - \frac{1}{b} = c$, $c - \frac{1}{c} = a$ then

find out the value of $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$

Solve

$$\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)^2 = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + 2\left(\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}\right)$$

$$a + \frac{1}{a^2} - 2 = b^2$$

$$b^2 + \frac{1}{b^2} - 2 = c^2$$

$$c^2 + \frac{1}{c^2} - 2 = a^2$$

$$0 = 6 + 2Q$$

$$-6 = 2Q$$

$$Q = -3 \text{ Ans}$$

6) If $a = 101$, $b = 102$ and $c = 103$, then

$$a^2 + b^2 + c^2 - ab - bc - ca = ?$$

$$\frac{1}{2} (a-b)^2 + (b-c)^2 + (c-a)^2$$

$$\frac{1}{2} (1 + 1 + 4)$$

3 Ans

Find the value of $(a^3 + b^3 + c^3 - 3abc)$

where $a = 335$, $b = 215$ and $c = 180$

$$(a+b+c) \frac{1}{2} (a-b)^2 + (b-c)^2 + (c-a)^2$$

$$365 \cancel{730} \times \frac{1}{2} \left((120)^2 + (35)^2 + (155)^2 \right)$$

$$5 \times 4 \times 5 \times 5$$

(*)

If

$$a = 90$$

$$b = 92$$

$$c = 95$$

$$a^3 + b^3 + c^3 - 3abc = ?$$

$$(a+b+c) \frac{1}{2} (a-b)^2 + (b-c)^2 + (c-a)^2$$

$$277 \times \frac{1}{2} (4+9+25)$$

$$277 \times \frac{1}{2} \times 38$$

$$19 \times 7$$

$$5263 \text{ Ans}$$

If $a^2 + b^2 + c^2 = ab + bc + ca$ find the

Value of $\frac{11a^4 + 13b^4 + 15c^4}{16a^2b^2 + 19b^2c^2 + 17c^2a^2}$

$$16a^2b^2 + 19b^2c^2 + 17c^2a^2$$

$$a = b = c = 1$$

$$\frac{11 + 13 + 15}{16 + 19 + 17}$$

$$= \frac{39}{52} = \frac{3}{4}$$

$$= \frac{3}{4} \text{ Ans}$$

$$a^2 + b^2 + c^2 = ab + bc + ca$$

$$\text{If } a = b = c$$

$$x^3 + y^3 + z^3 = 13$$

$$x + y + z = 1$$

$$xyz = 1$$

$$xy + yz + zx = ?$$

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + y + z)^2 - 3(xy + yz + zx)$$

$$10 = 1(1 - 3Q)$$

$$10 = 1 - 3Q$$

$$3Q = 9$$

$$Q = -3 \text{ Ans}$$

$$x^2 + y^2 + z^2 = 133$$

$$xy + yz + zx = 114$$

$$xyz = 216$$

$$x^3 + y^3 + z^3 = ?$$

Solve

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + y + z)^2 - 3(xy + yz + zx)$$

$$Q - 648 = 19(361 - 342)$$

$$Q = 648 = 19 \times 19$$

$$361$$

$$648$$

$$Q = 1009 \text{ Ans}$$

$$(x + y + z)^2 = 133 + 228$$

$$361$$

9

$$x+y+z = 11$$

$$x^2+y^2+z^2 = 133$$

$$x^3+y^3+z^3 = 881$$

$$\sqrt[3]{xyz} = ?$$

Solve

$$x^3+y^3+z^3-3xyz = (x+y+z)(x+y+z)^2-3(xy+yz+zx)$$

$$881-3Q = (11-318)$$

$$881-3Q = 11 \times 139 - 1529$$

$$-3Q = 1529 - 881$$

$$-3Q = 648$$

$$216$$

$$xyz = 216$$

$$\sqrt[3]{xyz} = -6 \text{ Ans}$$

10

$$p^3+q^3+r^3-3pqr = 4$$

$$a = q+r$$

$$b = r+p$$

$$c = p+q$$

$$a^3+b^3+c^3-3abc = ?$$

$$\rightarrow (a+b+c) \frac{1}{2} ((a-b)^2 + (b-c)^2 + (c-a)^2)$$

Solve

$$2(p+q+r) \cdot \frac{1}{2} \left((q-p)^2 + (r-q)^2 + (p-r)^2 \right)$$

$$2 \times p^3 + q^3 + r^3 - 3pqr$$

$$2 \times 4$$

$$= 8 \quad \underline{\text{Ans}}$$

15

$$\frac{x^3 + y^3 + z^3 - 3xyz}{(x-y)^2 + (y-z)^2 + (z-x)^2}$$

If $a+b+c = 6$, then $(a-1)^3 + (b-2)^3 + (c-3)^3$

Solve

$$(a-1) + (b-2) + (c-3) = 0$$

$$3(a-1)(b-2)(c-3) \quad \underline{\underline{\text{Ans}}}$$

$$x + y + z = 0$$

$$x^3 + y^3 + z^3 = 3xyz$$

If $a = 4.965$
 $b = 2.343$
 $c = 2.622$

$$a^3 + (-b)^3 + (-c)^3 - 3a(-b)(-c) = 0$$

$$a + (-b) + (-c) = 0$$

$$a^3 + (-b)^3 + (-c)^3 - 3a(-b)(-c) = 0$$

$$a - b - c = 0 \text{ Ans}$$

21) If $(5x-3)^3 + (2x+5)^3 + 27(4-3x)^3 = 9$

$$(3-5x)(2x+5)(3x-4), \text{ Find out}$$

$$(2x+1) = ?$$

$$(5x-3)^3 + (2x+5)^3 + (12-9x)^3 = 3(3-5x)(2x+5)(9x-12)$$

$$\begin{array}{r} 5x-3 \\ 2x+5 \\ -9x+12 \\ \hline -2x+14 = 0 \end{array}$$

$$x = 7$$

$$= 15 \text{ Ans}$$

2) If $(3x-1)^3 + (4x-3)^3 + (2x+1)^3 = 3$

$$(3x-1)(4x-3)(2x+1) \text{ and } x \neq \frac{1}{3} \text{ find}$$

out the value of x .

Solve

$$3x - 1 = 4x - 3$$

$$x = 2 \quad \underline{\text{Ans}}$$

(16)

$$\text{If } (x+y)^{\frac{1}{3}} + (y+z)^{\frac{1}{3}} + (z+x)^{\frac{1}{3}} = 0$$

$$\text{value} = x^3 + y^3 + z^3 = P$$

$$(x+y+z) = \frac{3}{2} \left((x+y)(y+z)(z+y) \right)^{\frac{1}{3}}$$

$$x^3 + y^3 + z^3 = (x+y+z)^3 - 3(x+y)(y+z)(z+x)$$

$$\frac{27}{8} (x+y)(y+z)(z+x) - 3(x+y)(y+z)(z+x)$$

$$(y+z)(z+x)$$

$$\frac{3}{8} (x+y)(y+z)(z+y) \quad \underline{\text{Ans}}$$

Square and Cube with different formates

$$\text{If } x + \frac{1}{x} = K$$

$$(*) \quad x^2 + \frac{1}{x^2} = K^2 - 2$$

$$(*) \quad x^3 + \frac{1}{x^3} = K^3 - 3K$$

$$x + \frac{1}{x} = K$$

$$x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} (K) = K^3$$

$$x^3 + \frac{1}{x^3} = K^3 - 3K$$

$$\text{If } \Rightarrow \quad x - \frac{1}{x} = K$$
$$x^2 + \frac{1}{x^2} = K^2 + 2$$
$$x^3 - \frac{1}{x^3} = K^3 + 3K$$

$$x + \frac{1}{x}$$

$$x^3 + \frac{1}{x^3}$$

2	→	2
3	→	18
4	→	52
5	→	110
6	→	198
7	→	302
8	→	488
9	→	702
10	→	970

$$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$x^6 - \frac{1}{x^6} = (x^3)^2 - \frac{1}{(x^3)^2} = \left(x^3 + \frac{1}{x^3}\right) \left(x^3 - \frac{1}{x^3}\right)$$

Factor $\Rightarrow$ $x^4 - \frac{1}{x^4} = (x^2)^2 - \frac{1}{(x^2)^2} = \left(x^2 + \frac{1}{x^2}\right) \left(x^2 - \frac{1}{x^2}\right)$

$$x^8 - \frac{1}{x^8} = (x^4)^2 - \frac{1}{(x^4)^2} = \left(x^4 + \frac{1}{x^4}\right) \left(x^4 - \frac{1}{x^4}\right)$$

Q $x + \frac{1}{x} = \sqrt{11}$

$x^3 + \frac{1}{x^3} = ?$

$8\sqrt{11}$ Ans

Q $x + \frac{1}{x} = \sqrt{7}$, $x^3 + \frac{1}{x^3} = ?$

$k^3 - 3k$

$(\sqrt{7})^3 - 3\sqrt{7}$

$7\sqrt{7} - 3\sqrt{7}$

$4\sqrt{7}$ Ans

Q $\Rightarrow$ $x - \frac{1}{x} = \sqrt{5}$, $x^3 - \frac{1}{x^3} = ?$

$8\sqrt{5}$ Ans

Type - 5

Square & Cube with different formant

①

If $\left(a + \frac{1}{a}\right) = 6$ then what is value

$$\frac{3}{4} \left(a^2 + \frac{1}{a^2}\right) ?$$

Solve

$$\frac{3}{4} \times 34 = \frac{102}{4}$$

$$= 22.5 \text{ Ans}$$

②

If $\left(a + \frac{1}{a}\right) = 5$, then what is value

$$\left(a^3 + \frac{1}{a^3}\right) ?$$

Solve

$$K^3 - 3K$$

$$125 - 15$$

$$110 \text{ Ans}$$

③

If $\left(x + \frac{1}{x}\right) = 10$, $\left(x^4 + \frac{1}{x^4}\right) ?$

$$x^2 + \frac{1}{x^2} = \because (10)^2 - 2 = 98$$

$$x^4 + \frac{1}{x^4} = (98)^2 - 2$$

$$= 9604 - 2$$

$$9602 \text{ Ans}$$

$(98)^2$ 9604

(4) If $\left(x + \frac{1}{x}\right) = 4$, $\left(x^5 + \frac{1}{x^5}\right) = ?$

Solve

(*) $x + \frac{1}{x} = K$

$$x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right) \left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$$

$$x^7 + \frac{1}{x^7} = \left(x^4 + \frac{1}{x^4}\right) \left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$$

$$(K^3 - 3K)(K^2 - 2) - K$$

$$(64 - 12)(14) - 4$$

$$52 \times 14 - 4$$

$$728 - 4$$

$$724 \text{ Ans}$$

(5) If $\left(x + \frac{1}{x}\right) = 7$, then $\left(x^6 + \frac{1}{x^6}\right) = ?$

$$x^3 + \frac{1}{x^3} = 322$$

$$x^6 + \frac{1}{x^6} = (322)^2 - 2$$

$$322 \times 322 - 2$$

$$4 - 2$$

$$= 103682 \text{ Ans}$$

$$\text{Did it} = 2$$

(6) If $\left(x + \frac{1}{x}\right) = 3$ then $\left(x^7 + \frac{1}{x^7}\right) = ?$

$$x^2 + \frac{1}{x^2} = 7$$

$$x^4 + \frac{1}{x^4} = 47$$

$$\left(x^4 + \frac{1}{x^4}\right) \left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$$

$$47 \times 18 - 3$$

$$846 - 3$$

(7)

$$\text{If } \left(x + \frac{1}{x}\right) = 5, \quad \left(x^8 + \frac{1}{x^8}\right) = ?$$

$$x^2 + \frac{1}{x^2} = 23$$

$$x^4 + \frac{1}{x^4} = 527$$

$$x^8 + \frac{1}{x^8} = (527)^2 - 2$$

$$527 \times 527 - 2$$

$$277729 - 2$$

$$277727 \text{ Ans}$$

(8)

$$\text{If } \left(y + \frac{1}{y}\right) = 7, \quad \left(y^6 + \frac{1}{y^6}\right) = ?$$

$$y^3 + \frac{1}{y^3} = 322$$

$$y^6 + \frac{1}{y^6} = (322)^2 - 2$$

$$103682 \text{ Ans}$$

(9)

$$\text{If } \left(x + \frac{1}{x}\right) = 5, \text{ \& } x > 1, \text{ then value } \left(x^8 - \frac{1}{x^8}\right)$$

$$\left(x^4\right)^2 - \left(\frac{1}{x^4}\right)^2 = \left(x^4 + \frac{1}{x^4}\right) \left(x^4 - \frac{1}{x^4}\right)$$

$$\left(x^4 + \frac{1}{x^4}\right) \left(x^2 + \frac{1}{x^2}\right) \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$527 \times 23 \times 5 \times \sqrt{21}$$

$$527 \times 115 \sqrt{21}$$

$$= 60605 \sqrt{21} \text{ Ans}$$

(22)

If $5x^{7/4} - 3x^{1/8} + 5 = 0$, then find out the value of $x^{1/4} + x^{1/4}$

$$5x^{\left(\frac{7}{8}\right)^2} - 3x^{\frac{1}{8}} + 5 = 0$$

$$\left(\div x^{\frac{7}{8}} + \frac{1}{x^{\frac{7}{8}}} \right) = \frac{3}{5}$$

$$k^2 = 2$$

$$\frac{9}{25} - 2$$

$$-\frac{41}{25} \text{ Ans}$$

(23)

If $x - 5\sqrt{x} - 1 = 0$, then $x^2 + \frac{1}{x^2}$ is

$$\sqrt{x} - \frac{1}{\sqrt{x}} = 5$$

$$x + \frac{1}{x} = 27$$

$$x^2 + \frac{1}{x^2} = 729$$

$$x^2 + \frac{1}{x^2} = 729 - 2 = 727 \text{ Ans}$$

(24)

If $x + \frac{1}{x} = \frac{\sqrt{3} + 1}{2}$, $x^4 + \frac{1}{x^4} = ?$

$$x^2 + \frac{1}{x^2} = \frac{4 + 2\sqrt{3}}{4} - 2 = \frac{-4 + 2\sqrt{3}}{4} = \frac{-2 + \sqrt{3}}{2}$$

$$x^4 + \frac{1}{x^4} = \left(\frac{-2 + \sqrt{3}}{2} \right)^2 - 2$$

$$7 - 4\sqrt{3} - 2 = \frac{1 - 4\sqrt{3}}{1} \text{ Ans}$$

25

If $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{15}$, then find out the value of $x\sqrt{x} + \frac{1}{x\sqrt{x}}$?

$$K^3 - 3K$$

$$(\sqrt{15})^3 - 3\sqrt{15}$$

$$15\sqrt{15} - 3\sqrt{15}$$

$$12\sqrt{15} \text{ Ans}$$

$$(\sqrt{x})^3 = x\sqrt{x}$$

26

If $\frac{3}{x^4} + \frac{1}{\left(\frac{3}{x^4} + 9\right)} = 0$, then find out the value of

$$\left(\frac{3}{x^4} + 9\right) + \frac{1}{\left(\frac{3}{x^4} + 9\right)} = 9$$

$$\left(\frac{3}{x^4} + 9\right)^2 - \frac{1}{\left(\frac{3}{x^4} + 9\right)^2} =$$

$$9 \times \sqrt{77}$$

Ans

$$\text{If } x + \frac{1}{x} = K$$

$$x^2 + \frac{1}{x^2} = K^2 - 2$$

$$x^3 + \frac{1}{x^3} = K^3 - 3K$$

$$ax + \frac{b}{x} = K$$

$$ax^2 + \left(\frac{b}{x}\right)^2 = K^2 - 2ab$$

$$(ax)^3 + \left(\frac{b}{x}\right)^3 = K^3 - 3Kab$$

(27)

$$\text{If } x + \frac{1}{9x} = 3, \quad 27x^3 + \frac{1}{27x^3} = ?$$

$$(3x)^3 + \frac{1}{(3x)^3} = ?$$

$$k^3 - 3k$$

$$729 - 27$$

$$702 \text{ Ans}$$

$$3x + \frac{1}{3x} = 9$$

(28)

$$\text{If } x + \frac{1}{x+2} = 6+2 \quad \left| \quad (x+2)^3 + \frac{1}{(x+2)^3} = ? \right.$$

$$(x+2) + \frac{1}{(x+2)} = 8$$

$$k^3 - 3k$$

$$512 - 24$$

$$488 \text{ Ans}$$

(29)

$$\text{If } \frac{4x \times \frac{10}{4}}{5x \times \frac{1}{4}} = 6, \quad 1000x^3 + \frac{1}{8x^3} = ?$$

$$\frac{10}{4} \quad 10x - \frac{1}{2x} = 15$$

$$a = 10$$

$$b = \frac{1}{2}$$

$$ax - \frac{b}{x} = k$$

$$(10x^3) + \frac{1}{(2x)^3}$$

$$k^3 + 3kAB$$

$$3375 + 3 \times 15 \times 10 \times \frac{1}{2}$$

$$3375 + 225$$

$$= 3600 \text{ Ans}$$

Imp

(30)

$$\text{If } x + \frac{1}{16x} = 3, \quad 16x^3 + \frac{1}{256x^3} = ?$$

$$2x + \frac{1}{8x} = 6$$

$$2 \left(8x^3 + \frac{1}{512x^3} \right)$$

$$2 \left((2x)^3 + \frac{1}{(8x)^3} \right)$$

$$2(216 - 3 \times 6 \times 2 \times 1)$$

$$432 - 9 = 423 \text{ Ans}$$

32

If $a + a^2 + a^3 = 1$, then find $a^3 + \frac{1}{a}$ is

$$\frac{a^3 + a^1 + a^2 + a^3}{a}$$

$$\frac{a^3 + 1 + a + a^2}{a}$$

2 Ans

33

If $a^3 + 3a^2 + 9a = 1$, then find $a^3 + \frac{3}{a}$ is

$$\begin{array}{l} \text{is} \\ \times 3 \\ \hline 3a^3 + 9a^2 + 27a = 3 \end{array}$$

$$\frac{a^3 + 3a^3 + 9a^2 + 27a}{a}$$

$$\frac{a^3 + 3a^2 + 9a + 27}{1 + 27 = 28}$$

Ans

If $a + a^2 + a^3 + a^4 - 2 = 0$, then find $a^4 + \frac{2}{a}$ is

$$\frac{a^4 + a^1 + a^2 + a^3 + a^4}{a}$$

$$\frac{a^4 + 1 + a^1 + a^2 + a^3}{2 + 1}$$

Ans

Concept

(35)

(*)

$$a^5 + a^4 + a^3 + a^2 + a + 1 = 0$$

$$a^6 = 1$$

(*)

$$a^7 + a^6 + a^5 + a^4 + a^3 + a^2 + a + 1 = 0$$

$$a^8 = 1$$

(35)

If $a^4 + a^3 + a^2 + a + 1 = 0$, then

what is the value of $a^{15} + a^{10} - 2$.

$$a^5 = 1$$

$$(a^5)^3 + (a^5)^2 - 2$$

$$1 + 1 - 2$$

$$= 0 \text{ Ans}$$

(36)

If $a^5 + a^4 + a^3 + a^2 + a + 1 = 0$,

then find the value of $a^{36} + a^{18} + 1$

$$a^6 = 1$$

$$(a^6)^2 + (a^6)^3 + 1$$

$$1 + 1 + 1$$

$$(3) \text{ Ans}$$

(37)

If $x^2 - 12x + 33 = 0$, then find

$$(x-4)^4 + \frac{1}{(x-4)^4}$$

Solve

Imp

$$x - 4 = m$$

$$x = m + 4$$

$$(m+4)^2 - 12(m+4) + 33 = 0$$

$$m^2 + 16 + 8m - 12m - 48 + 33 = 0$$

$$m^2 + 16 + 8m - 12m - 48 + 33 = 0$$

$$m^2 - 4m + 1 = 0$$

$$m + \frac{1}{m} = 4$$

$$m = 194 \text{ Ans}$$

$$m^4 + \frac{1}{m^4}$$

38

If $x^2 + x = 5$, $\frac{1}{(x+3)^3} - \frac{1}{(x+3)^3} = ?$

$$x + 3 = m$$

$$x = m - 3$$

$$(m-3)^2 + (m-3) = 5$$

$$m^2 + 9 - 6m + m - 3 = 5$$

$$m^2 - 5m + 1 = 0$$

$$m + \frac{1}{m} = 5$$

$$m^3 + \frac{1}{m^3}$$

$$K^3 - 3K$$

$$125 - 15$$

$$110 \text{ Ans}$$

$\sqrt{x + \frac{1}{\sqrt{x}}} = 3, x > 0$ find $x^2(x^2 + 47)$

$$x + \frac{1}{x} = 7$$

$$x^2 + \frac{1}{x^2} = 47$$

$$x^2 x - \frac{1}{x^2} = -1 \text{ Ans}$$

(40)

If $x^3 - 6x^2 + 35 = 8(x-2)$, find

$$x^2 + \frac{3}{x-6} = ?$$

$$\begin{array}{r|l} \frac{x^3 - 6x^2 + 3}{x-6} & \begin{array}{l} 8x - 16 - 35 \\ x^3 - 6x^2 = 8x - 51 \end{array} \\ \frac{8x - 51 + 3}{x-6} = \frac{8x - 48}{x-6} & \\ \hline = 8 \text{ Ans} & \end{array}$$

(41)

If $x^2 - 38x + 96 = 0$, find $(x-5) - \frac{69}{x-5}$

$$\begin{array}{r} (x-5)^2 - 69 \\ \hline x-5 \\ \hline = x^2 + 25 - 10x - 69 \\ \hline x-5 \\ \hline = x^2 \end{array}$$

$$x-5 = m$$

$$x = m+5$$

$$(m+5)^2 - 38(m+5) + 96 = 0$$

$$m^2 + 25 + 10m - 38m - 190 + 96 = 0$$

$$m^2 - 28m - 69 = 0$$

$$m - \frac{69}{m} = 28 \text{ Ans}$$

42

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If $x^8 - 32x^4 + 256 = 0$, then

$x^4 = p$

$x^4 = 16$

$x = 2$

Positive value of $\left(x^2 + \frac{1}{x^2}\right)$ is ?

$p^2 - 32p + 256 = 0$

$(p - 16)^2 = 0$

$p - 16 = 0$

$p = 16$

$4 + \frac{1}{4}$

$\frac{17}{4} = 4.25 \text{ Ans}$

43

If $a - \frac{24}{a} = 5$, ($a > 0$), find $a^2 + \frac{64}{a^2}$?

$a = 8$

$64 + \frac{64}{64}$

65 Ans

44

If $x^8 - 1442x^4 + 1 = 0$, then the possible value of $x - \frac{1}{x}$ is ?

$x^4 = p$

$p^2 - 1442p + 1 = 0$

$p + \frac{1}{p} =$

$\left(x - \frac{1}{x}\right)^2 = 36$

$x - \frac{1}{x} = 6 \text{ Ans}$

$x^4 + \frac{1}{x^4} = 1442$

$\left(x^2 + \frac{1}{x^2}\right)^2 = \sqrt{1444}$

$x^2 + \frac{1}{x^2} = 38$

$\begin{array}{r} 1444 \\ \sqrt{} \\ 38 \end{array}$

- (45) If $8K^6 - 31K^3 - 4 = 0$ find negative values of $K + \frac{1}{K}$

Solve

$$K^3 = p \text{ let}$$

$$8p^2 - 31p - 4 = 0$$

$$-32$$

$$\begin{array}{r} -32 \quad 1 \\ + 32 \quad -1 \\ \hline 8 \quad 8 \end{array}$$

$$p = 4, -\frac{1}{8}$$

$$K = \left(-\frac{1}{2}\right)$$

$$-\frac{1}{2} - 2$$

$$-\frac{5}{2} \text{ Ans}$$

- (46) If $(x-a)(x-b) = 1$ & $a-b+3=0$ find

$$(x-a)^3 - \frac{1}{(x-a)^3}$$

Solve

$$x^3 - \frac{1}{x^3}$$

$$K^3 + 3K$$

$$27 + 9$$

$$= 36 \text{ Ans}$$

If $(x+2)(x-1) = 1$, find $(x+2)^3 - \frac{1}{(x+2)^3}$

$$x+1 = \frac{1}{x+2}$$

$$x+2 = \frac{1}{x+2} + 3$$

$$K^3 + 3K$$

$$27 + 9$$

$$\boxed{36} \text{ Ans}$$

Type - 6

Three identity

If $(*) \left[x + \frac{1}{x} = 1, x^3 = -1 \right]$

$(*) \left[x + \frac{1}{x} = \sqrt{3}, x^6 = -1 \right]$

$(*) \left[x + \frac{1}{x} = \sqrt{2}, x^4 = -1 \right]$

sq $\left[\begin{aligned} x + \frac{1}{x} &= 1 \\ x^2 + \frac{1}{x^2} &= -1 \\ x^3 + \frac{1}{x^3} &= -x \end{aligned} \right]$

$$x^3 = -x - \frac{1}{x}$$

$$\begin{aligned} & - \left(x + \frac{1}{x} \right) \\ & \boxed{x^3 = -1} \end{aligned}$$

If $x + \frac{1}{x} = 1$ then find out the given function $\Rightarrow x^3 = -1$

$(*) \quad x^{18} + \frac{1}{x^{18}} = (x^3)^6 + \frac{1}{(x^3)^6}$

$$1 + 1$$

$\boxed{2} \text{ Ans}$

$$(b) \quad x^{17} + \frac{1}{x^{17}} = ?$$

$$\frac{x^{18}}{x} + \frac{x}{x^{18}} = \frac{(x^3)^6}{x} + \frac{x}{(x^3)^6} = \frac{1}{x} + x = 1 \quad \underline{\text{Ans}}$$

$$(c) \quad x^{14} + \frac{1}{x^{14}} = \frac{x^{15}}{x} + \frac{x}{x^{15}}$$

$$\frac{(x^3)^5}{x} + \frac{x}{(x^3)^5} = \left(\frac{1}{x} + x \right) = -1 \quad \underline{\text{Ans}}$$

$$(2) \quad x + \frac{1}{x} = \sqrt{3} \quad (Q) \quad x^{46} + x^{40} = ?$$

$$(Q) \quad x + \frac{1}{x} = 1 \quad x^{40} (x^6 + 1) = 0 \quad \underline{\text{Ans}}$$

$$x^{33} + x^{30} = ?$$

$$x^{30} (x^3 + 1)$$

$$= 0 \quad \underline{\text{Ans}}$$

$$x^3 = -1$$

$$x + \frac{1}{x} = \sqrt{3}, \text{ then } x^6 = -1$$

$$x^{48} + x^{42} + x^{36} + x^{30} + x^{24} + x^{18} + x^{12} + x^6 + 5$$

$$(x^{18} + 5$$

$$(x^6)^3 + 5$$

$$-1 + 5$$

③

If $\sqrt{x} - \frac{1}{\sqrt{x}} = x\sqrt{x}$, find $x^{12} + \frac{1}{x^{12}}$

$$1 - \frac{1}{x} = x$$

$$x + \frac{1}{x} = 1$$

$$x^3 = -1 \quad \underline{\text{Ans}}$$

$$(x^3)^4 + \frac{1}{(x^3)^4}$$

$$1+1 = 2 \quad \underline{\text{Ans}}$$

④

If $x + \frac{1}{x} = \sqrt{3}$, $x^{210} + \frac{1}{x^{210}} = ?$

Sol

$$x^6 = 1$$

$$(x^6)^{35} + \frac{1}{(x^6)^{35}}$$

$$-1-1 = -2 \quad \underline{\text{Ans}}$$

⑤

If $x + \frac{1}{x} = 1$, $x^{17} + \frac{1}{x^{17}} = ?$

Sol

$$x^3 = -1$$

$$\frac{x^{18}}{x} + \frac{x}{x^{18}}$$

$$\frac{(x^3)^6}{x} + \frac{x}{(x^3)^6} = \frac{1}{x} + x = 1$$

⑥

If $x + \frac{1}{x} = \sqrt{3}$, $x^{206} + x^{200} = ?$

$$x^6 = -1$$

$$x^{200}(x^6 + 1)$$

$$0 \quad \underline{\text{Ans}}$$

If $x + \frac{1}{x} = \sqrt{3}$, $x^{46} +$

Q If $x + \frac{1}{x} = 1$, $x^{23} + x^{20} = ?$
 $x^3 = -1$ $x^{20}(x^3 + 1)$
Ans

Q If $x + \frac{1}{x} = \sqrt{2}$, $x^{1003} + \frac{1}{x^{1003}} = ?$

$x^4 = -1$

$x^{100} \left(x^3 + \frac{1}{x^3} \right)$
 $(x^4)^{250} x^3 + \frac{1}{(x^4)^{250} x^3}$

Trick

$\frac{x^{1004}}{x} + \frac{x}{x^{1004}}$
 $\frac{(x^4)^{251}}{x} + \frac{x}{(x^4)^{251}}$

$x^3 + \frac{1}{x^3}$
 $= 1^3 - 3 \cdot 1$
 $= 2\sqrt{2} - 3\sqrt{2} = -\sqrt{2}$ Ans

$-\left(\frac{1}{x} + x\right) = -\sqrt{2}$ Ans

Q If $x + \frac{1}{x} = 1$, $x^{103} + x^{100} + x^{93} + x^{90} + x^{83} + x^{80} + x^6 + 1 = ?$

$(x^3 - 1)$

$0 + 0 + 0 + 0 + 0 + 0 + 1 + 1$

$1 + 1$
 $= 2$ Ans

If $x + \frac{1}{x} = 1$

sq $x^2 + \frac{1}{x^2} = -1$

xx $x^3 + \frac{1}{x} = -x$

$x^3 = x^{-1}$

$x^3 = -\left(x + \frac{1}{x}\right)$

$x^3 = -1$ Ans

(4) If $x^2 + x = -1$, find $(1+x^{16}+x^{17})(1-x^{16}+x^{17})$
 $(1+x+x^2)(1-x+x^2)$
 $x + \frac{1}{x} = -1$
 $x^3 = 1$ $x^{15} = 1$
 x^2

$$(-x^2-x^2)(-x-x)$$

$$-2x^2x - 2x$$

$$4x^3 \text{ Ans}$$

(5) If $\frac{x}{5\sqrt{6}} + 1 = \frac{-5\sqrt{6}}{x}$,

$$x^5 - 750\sqrt{6}x^2 + 1 = 1 \text{ Ans}$$

$$\frac{x}{5\sqrt{6}} + \frac{5\sqrt{6}}{x} = -1$$

$$\left(\frac{x}{5\sqrt{6}}\right)^3 = 1 \quad \frac{x^3}{750\sqrt{6}} = 1$$

$$x^3 = 750\sqrt{6}$$

$$\text{Ans} = 1 \text{ Ans}$$

(6) If $\left(\frac{x}{3}\right) + \left(\frac{3}{x}\right) + \frac{3}{x} = 1$, then x^3 ?
 $\downarrow$
 -27

$$\left(\frac{x}{3}\right)^3 = -1$$

$$x^3 = -27 \text{ Ans}$$

⑧ If $\frac{a}{4} + 1 = \frac{-4}{a}$, then $a^5 - \frac{64a^2 + 1}{a^3}$?

$$\frac{a}{4} + \frac{4}{a} = -1$$

1 Ans

$$\left(\frac{a}{4}\right)^3 = 1$$

$$a^3 = 64$$

⑨ If $x(x-1) = -1$, then $x^4 - 6x^3 + 9x^2 - 8x + 11$

$$x-1 = -\frac{1}{x}$$

$$x + \frac{1}{x} = 1$$

$$x^3 = -1$$

$$-x + 6 + \frac{9x^3}{x} - 8x + 11$$

$$-9x - \frac{9}{x} + 17$$

$$-9\left(x + \frac{1}{x}\right) + 17$$

$$-9 + 17 = 8 \text{ Ans}$$

⑩ If $7a + \frac{7}{a} = \sqrt{98}$, Find $a^{17} + \frac{1}{a^{25}} = ?$

$$7\left(a + \frac{1}{a}\right) = 7\sqrt{2}$$

$$a + \frac{1}{a} = \sqrt{2}$$

$$a^4 = -1$$

$$\frac{(a^4)^4 a^1 + 1}{(a^4)^6 a^1}$$

$$a + \frac{1}{a}$$

$$\sqrt{2} \text{ Ans}$$

11

If $x + \frac{1}{x} = -1$,

$x^{15} + \frac{1}{x^{15}}$ will be

If $x + \frac{1}{x} = -1$

$x^3 = 1$

$(x^3)^5 + \frac{1}{(x^3)^5}$

$1 + 1$

$\boxed{2}$ Ans

6

If $x^2 - 7x + 13 = 0$, then $(x-3)^{22} + \frac{1}{(x-3)^{22}}$

$(m+3)^2 - 7(m+3) + 13 = 0$

$\boxed{\begin{matrix} x-3 = m \\ x = m+3 \end{matrix}}$

$m^2 + 9 + 6m - 7m - 21 + 13 = 0$

$m^2 - m + 1 = 0$

$m + \frac{1}{m} = 1$

$m^3 = -1$

$m^{22} + \frac{1}{m^{22}}$

$(m^3)^7 m + \frac{1}{(m^3)^7 m}$

$= \left(m + \frac{1}{m}\right)$

$= 1$ Ans

Type - 2

Concept of two factor

① $x^4 + y^4 + x^2 y^2 = (x^2 + y^2 + xy)(x^2 + y^2 - xy)$

② $x^2 + y^2 + xy = (x + y + \sqrt{xy})(x + y - \sqrt{xy})$

③ $x^4 + y^4 = (x^2 + y^2 + \sqrt{2xy})(x^2 + y^2 - \sqrt{2xy})$

④ $x^4 + y^4 - 3x^2 y^2 = (x^2 - y^2 + xy)(x^2 - y^2 - xy)$

⑤ $16x^4 + 36x^2 y^2 + 48y^4 = (4x^2 + 9y^2 - 6xy)(4x^2 + 9y^2 + 6xy)$

Q If $x^4 + y^4 + x^2 y^2 = 100$

\$ $x^2 + y^2 + xy = 20$

$x^2 + y^2 - xy = 5$ Ans

Q $x^2 + y^2 + xy = p$

$x^2 + y^2 - xy = q$

$x^2 + y^2 = \frac{p+q}{2}$

$xy = \frac{p-q}{2}$

①

If $a^4 + b^4 + 2a^2b^2 = 133$, and $a^2 + b^2 - ab = 19$ then value of $(ab) = ?$

$$a^2 + b^2 + ab = 7p$$

$$ab = \frac{7 - 19}{2}$$

$$= -6 \text{ Ans}$$

②

If $x^4 + x^2y^2 + y^4 = \frac{21}{256}$ and $x^2 - xy + y^2 = \frac{3}{16}$

(i) then $2(x^2 + y^2) = ? \frac{5}{16}$

(ii) $xy = \frac{2}{16}$

(iii) $x^4 + y^4 = \frac{17}{256}$

(iv) $x + y =$

$$\frac{\frac{21}{256}}{\frac{16}{16}} \times \frac{\frac{16}{3}}{\frac{16}{3}} \sqrt{\frac{5}{16} \times \frac{4}{16}} = \frac{3}{4} \text{ Ans}$$

③

If $x^4 + y^4 - 3x^2y^2 = 12$, $x^2 - y^2 + xy = 6$
Find $x^2 - y^2$

$$x^2 - y^2 + xy = 6$$

$$x^2 - y^2 - xy = 2$$

$$2(x^2 - y^2) = 8 \text{ Ans}$$

(4) If $1 + 9x^2 + 81x^4 = 256$, and $1 - 3x + 9x^2 = 32$,
then the value $1 + 3x + 9x^2 = \frac{256}{32} = 8$ Ans

(5) If $a + b - ab = 39$, $\frac{a - b - \sqrt{ab}}{a^2 + b^2 + \sqrt{ab}} = ?$

$$\frac{a + b + \sqrt{ab}}{(a + b + \sqrt{ab})(a + b - \sqrt{ab})}$$

$\frac{1}{39}$ Ans

(6) If $a^2 + b^2 = x$ and $ab = y$, then the value
of $\frac{a^4 + b^4}{a^2 - \sqrt{2ab} + b^2} = ?$

$$\frac{(a^2 + b^2 + \sqrt{2ab})(a^2 + b^2 - \sqrt{2ab})}{a^2 + b^2 - \sqrt{2ab}}$$

$x\sqrt{25}$ Ans

Type - 3

Symmetric function / Cyclic function

1) If $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, then

$$\left[\left(\frac{1}{z-1} \right) \left(\frac{1}{x} + \frac{1}{y} \right) \right] + \left[\left(\frac{1}{y-1} \right) \left(\frac{1}{x} + \frac{1}{z} \right) \right] + \left[\left(\frac{1}{x-1} \right) \left(\frac{1}{y} + \frac{1}{z} \right) \right] = ?$$

Let $x=y=z$ | $\frac{1}{2} \left(\frac{2}{3} \right) \times 3$
 $\frac{3}{x} = 1$

$x=3$

Ans

If $\frac{a}{1-2a} + \frac{b}{1-2b} + \frac{c}{1-2c} = \frac{1}{2}$, find

$$\left[\frac{1}{1-2a} \right] + \left[\frac{1}{1-2b} \right] + \left[\frac{1}{1-2c} \right]$$

$a=b=c$

$\frac{a}{1-2a} \times 3 = \frac{1}{2}$ A

$\frac{1}{1-\frac{1}{4}} \times 3$

$6a = 1 - 2a$

$2a =$

$8a = 1$

$a = \frac{1}{8}$

$\frac{1}{1-\frac{1}{4}} \times 3 = 4$ Ans

(3) If $x = \frac{a-b}{a+b}$, $y = \frac{b-c}{b+c}$, $z = \frac{c-a}{c+a}$

$$\frac{(1-x)(1-y)(1-z)}{(1+x)(1+y)(1+z)} = ? = 1 \text{ Ans}$$

$a = b = c$
 $x = 0$
 $y = 0$
 $z = 0$

(4) If $a+b+c = abc$, then

$$\frac{(1-a^2)(1-b^2)}{ab} + \frac{(1-b^2)(1-c^2)}{bc} + \frac{(1-c^2)(1-a^2)}{ca}$$

Let

$a = b = c$
 $3a = a^3$
 $a^2 = 3$

$$\frac{(1-3)(1-3)}{3} \times 3$$

$$-2 \times -2$$

(4) Ans

(5) If $abc = 1$ then

$$\frac{a+1}{ab+a+1} + \frac{b+1}{bc+b+1} + \frac{c+1}{ca+c+1} = ?$$

$a = b = c = 1$ | $\frac{2}{2} \times 3 = 3$ Ans

5

If $xy + yz + zx = 1$

$$\frac{(x+y)}{1-xy} + \frac{(y+z)}{1-yz} + \frac{(z+x)}{1-zx} = ?$$

Solve

$$\frac{x+y}{z(x+y)} + \frac{1}{x} + \frac{1}{y}$$

$$\frac{1}{z} + \frac{1}{x} + \frac{1}{y} = \frac{xy + yz + zx}{xyz} = \frac{1}{xyz} \text{ Ans}$$

9

If $a^2 = b+c, b^2 = c+a$ and $c^2 = a+b$

then $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = ?$

Solve

Put

$$a = b = c$$

$$a^2 = 2a$$

$$a = 2$$

$$\frac{1}{3} \times 3 = 1 \text{ Ans}$$

10

If $\frac{(x+a)}{b+c} + \frac{x+b}{c+a} + \frac{x+c}{a+b} = -3$, then $x = ?$

Solve

$$a = b = c$$

$$\frac{x+a}{2a} \times 3 = -3$$

$$x+a-2a$$

$$x = -3a \text{ Ans}$$

(11)

$$\frac{(x-a)^2}{b+c} + \frac{(x-b)^2}{c+a} + \frac{(x-c)^2}{a+b} = 4(a+b+c),$$

then x ?

Let $a=b=c$

$$\frac{x-a^2}{2a} \times 3 = 4 \times 3a$$

$$x-a^2 = 8a^2$$

$$x = 9a^2 \quad \underline{\text{Ans}}$$

(12)

$$\text{If } \frac{x+a^2+2c^2}{b+c} + \frac{x+b^2+2a^2}{c+a} + \frac{x+c^2+2b^2}{a+b} = 0 \text{ then } x.$$

(8)

$$\text{If } x^2 + y^2 = z + 1$$

$$y^2 + z^2 = x + 1$$

$$z^2 + x^2 = y + 1$$

$$xyz = ?$$

Solve

$$x = y = z$$

$$2x^2 = x + 1$$

$$2x^2 - x - 1 = 0$$

$$x = 1, -\frac{1}{2} \quad \underline{\text{Ans}}$$

Inverse and root function

$$A \cdot B = 1$$

$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} + \frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{2(a+b)}{a-b}$$

$$a \cdot b = 1$$

$$b = \frac{1}{a}$$

$$x = 3 - 2\sqrt{2}$$

$$\frac{1}{x} = ?$$

$$\frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$\frac{3-2\sqrt{2}}{1}$$

1) If $x = 3+2\sqrt{2}$ find $\sqrt{x} - \frac{1}{\sqrt{x}}$

$$\frac{1}{x} = 3-2\sqrt{2}$$

$$x + \frac{1}{x} = 6$$

$$\sqrt{\left(x - \frac{1}{\sqrt{x}}\right)^2}$$

$$\sqrt{x + \frac{1}{x} - 2}$$

(2) If $x = 3 + 2\sqrt{2}$, find $x^2 + \frac{1}{x^2}$

$$\frac{1}{x} = 3 - 2\sqrt{2}$$

$$x + \frac{1}{x} = 6$$

$$x^2 + \frac{1}{x^2} = 6^2 - 2$$

$$36 - 2$$

$$34 \text{ Ans}$$

(3) If $x = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ and y is the reciprocal

of x , then what is the value of $(x^3 + y^3) = ?$

Solve

$$x + \frac{1}{x} = \frac{2(a+b)}{a-b} = \frac{2(8)}{2}$$

$$x + \frac{1}{x} = 8 \text{ Ans}$$

(4) If $a = \frac{\sqrt{5}+2}{\sqrt{5}-2}$, $b = \frac{\sqrt{5}-2}{\sqrt{5}+2}$ find

$$2a^2 + 2b^2 - 5ab$$

Solve

$$a = \frac{\sqrt{5} + \sqrt{4}}{\sqrt{5} - \sqrt{4}}$$

$$a + \frac{1}{a} = \frac{2(9)}{1} = 18$$

$$2\left(a^2 + \frac{1}{a^2}\right) - 5$$

$$2(322) - 5$$

$$644 - 5$$

$$639 \text{ Ans}$$

(5)

If $x = \sqrt{10} + \sqrt{11}$, $y = \sqrt{10} - \sqrt{11}$, then

Value of $7x^2 - 50xy + 7y^2 = ?$

$$\begin{array}{l|l} \begin{array}{l} x+y=2\sqrt{10} \\ xy=-1 \\ \rightarrow x^2+y^2-2=40 \end{array} & \begin{array}{l} 7(x^2+y^2) - 50xy \\ 7(42) + 50 \\ 294 + 50 \\ = 344 \text{ Ans} \end{array} \end{array}$$

(6)

If $a = \sqrt{8} - \sqrt{7}$ and $a = \frac{1}{b^2}$, then

$\frac{a^2 + b^2 - 3ab}{a^2 + ab + b^2}$ is equal to = ?

$$\begin{array}{l|l} \begin{array}{l} \frac{1}{a} = \sqrt{8} + \sqrt{7} \\ \hline a + \frac{1}{a} = 2\sqrt{8} \end{array} & \begin{array}{l} 30 - 3 \\ \hline 30 + 1 \\ 27 \\ \hline 31 \text{ Ans} \end{array} \end{array}$$

(7)

If $x = 3 + 2\sqrt{2}$, then the value of

$x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

$$\begin{array}{l} \left(x^3 + \frac{1}{x^3}\right) + \left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) + 1 \\ 198 + 34 + 6 + 1 \\ 239 \text{ Ans} \end{array}$$

$$\begin{array}{l} x = 3 + 2\sqrt{2} \\ \frac{1}{x} = 3 - 2\sqrt{2} \\ \hline x + \frac{1}{x} = 6 \end{array}$$

Q9

$$x = \sqrt{5} + 1 \text{ and } y = \sqrt{5} - 1$$

$$\text{find } \frac{x^2}{y^2} + \frac{y^2}{x^2} + 4\left(\frac{x}{y} + \frac{y}{x}\right) + 6$$

$$p = \frac{x}{y} = \frac{\sqrt{5} + 1}{\sqrt{5} - 1}$$

$$p + \frac{1}{p} = \frac{2(6)}{4} = 3$$

$$p^2 + \frac{1}{p^2} + 4\left(p + \frac{1}{p}\right)$$

$$7 + 12 + 6$$

$$= 25 \text{ Ans}$$

Q10

$$\text{If } x = 1 + \sqrt{2} + \sqrt{3} \text{ and } y = 1 + \sqrt{2} - \sqrt{3},$$

$$\text{then } \frac{x^2 + 4xy + y^2}{x+y} = ?$$

$$\frac{(x+y)^2 + 2xy}{x+y}$$

$$x+y = 2 + 2\sqrt{2}$$

$$xy = 3 + 2\sqrt{2} - 3$$

$$\frac{12 + 8\sqrt{2} + 4\sqrt{2}}{2 + 2\sqrt{2}}$$

$$= \frac{12 + 12\sqrt{2}}{2 + 2\sqrt{2}} = 6 \text{ Ans}$$

Type - 10 (Concept Square root)

(#)

$$\sqrt{7+4\sqrt{3}}$$

$$\sqrt{7+2 \times 2 \sqrt{3}}$$

$\downarrow \quad \downarrow$
 $a \quad b$

$$\sqrt{(2+\sqrt{3})^2} = 2+\sqrt{3} \text{ Ans}$$

(#)

$$\sqrt{13-4\sqrt{3}}$$

$$\sqrt{13-2 \times 2 \sqrt{3} \times 1}$$

$\quad \quad \quad \downarrow \quad \downarrow$
 $\quad \quad \quad a \quad b$

$$\sqrt{(2\sqrt{3}-1)^2} = 2\sqrt{3}-1 \text{ Ans}$$

(#)

$$\sqrt{38+5\sqrt{3}}$$

$$\sqrt{76+2 \times 5\sqrt{3} \times 1}$$

$$\frac{\sqrt{(5\sqrt{3}+1)^2}}{2} = \frac{5\sqrt{3}+1}{\sqrt{2}} \text{ Ans}$$

2

the value of $\sqrt{28+10\sqrt{3}} - \sqrt{7-4\sqrt{3}}$ is

Closest to $\frac{2 \times 5\sqrt{3}}{2 \times 2\sqrt{3}}$

Solve

$$(5+\sqrt{3}) - (2-\sqrt{3})$$

$$5+\sqrt{3}-2+\sqrt{3}$$

$$3+2\sqrt{3}$$

$$3+2 \times 1.732$$

(3)

$$\text{If } \sqrt{86 - 60\sqrt{2}} = a - b\sqrt{2}, \text{ then find } \sqrt{a^2 + b^2} = ?$$

$$\begin{array}{l} \downarrow \\ 2 \times 6 \times 5\sqrt{2} \\ b^2 a^2 \end{array} \quad \begin{array}{l} 36 + 25 \\ \sqrt{61} \end{array}$$

$$5\sqrt{2} - 6 = a - b\sqrt{2}$$

$$b = -5$$

$$a = -6 = 7.8 \text{ Ans}$$

(4)

$$\sqrt{10 - 2\sqrt{21}} + \sqrt{8 + 2\sqrt{15}} = \sqrt{a} + \sqrt{b}$$

where a and b are positive integers
then the value of $\sqrt{ab}$ is closest to

$$\sqrt{7} \times \sqrt{3}$$

$$\sqrt{5} \times \sqrt{3}$$

$$\sqrt{7} - \sqrt{3} + \sqrt{5} + \sqrt{3}$$

$$\sqrt{7} + \sqrt{5} = \sqrt{a} + \sqrt{b}$$

(5)

$$\text{If } \frac{\sqrt{26 - 7\sqrt{3}}}{\sqrt{14 + 5\sqrt{3}}} = \frac{b + a\sqrt{3}}{11}$$

value of $\sqrt{b-a}$?

$$b = 19$$

$$a = -6$$

$$\sqrt{19 - (-6)}$$

$$= 5 \text{ Ans}$$

$$\frac{\sqrt{52 - 2 \times 7\sqrt{3}}}{\sqrt{28 + 2 \times 5\sqrt{3}}} = \frac{7 - \sqrt{3}}{5 + \sqrt{3}} \times \frac{5 - \sqrt{3}}{5 - \sqrt{3}}$$

$$\frac{35 - 7\sqrt{3} - 5\sqrt{3} + 3}{22}$$

$$\frac{38 - 12\sqrt{3}}{11} = \frac{19 - 6\sqrt{3}}{11}$$

⑥ If $\sqrt{11-3\sqrt{8}} = a+b\sqrt{2}$, then what is the value of $(2a+3b)$

$$\sqrt{11-2\sqrt{2} \times 3}$$

$$3-\sqrt{2} = a+b\sqrt{2}$$

$$a=3$$

$$b=1$$

$$6-3$$

$$= 3 \text{ Ans}$$

⑦ If $x = \frac{\sqrt{3}}{2}$,

$$\sqrt{1+x} = ?$$

$$\sqrt{1+\frac{\sqrt{3}}{2}}$$

$$\sqrt{\frac{2+\sqrt{3}}{2}} = \sqrt{\frac{4+2\sqrt{3} \times 1}{4 \times 2}} = \frac{\sqrt{3+1}}{\sqrt{2}}$$

(*)

$$\text{If } x = \frac{\sqrt{3}}{2}$$

(*)

$$\sqrt{1+x} = \frac{\sqrt{3+1}}{2}$$

$$\sqrt{1-x} = \frac{\sqrt{3-1}}{2}$$

If $x = \sqrt{1+\frac{\sqrt{3}}{2}} - \sqrt{1-\frac{\sqrt{3}}{2}}$ the value

$$\sqrt{\frac{3-x}{1+x}} = ?$$

$$x = \left(\frac{\sqrt{3}+1}{2} \right) - \left(\frac{\sqrt{3}-1}{2} \right)$$

$$x = 1$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$\frac{(\sqrt{3}-1)^2}{2}$$

$$\frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

$$= 2 - 1.732$$

$$= 0.27 \text{ Ans}$$

⑧ If $x = 5 - \sqrt{2}$ then find out the value of $\sqrt{x}$

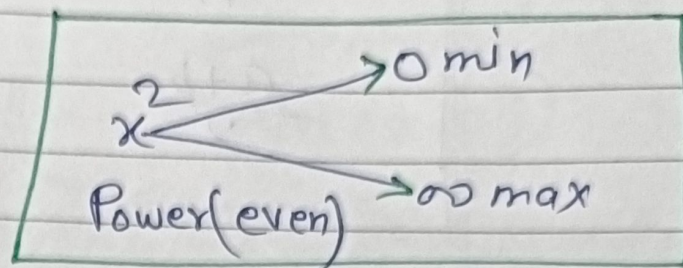
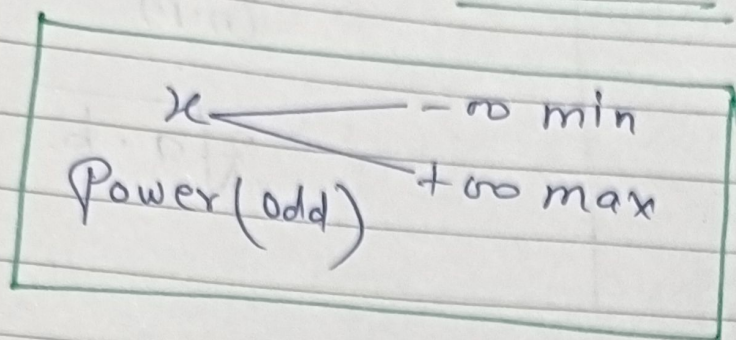
$$\sqrt{32-2x-\sqrt{2}}$$

$$\sqrt{x} = \sqrt{5-\sqrt{2}}$$

$$\frac{\sqrt{10-2\sqrt{2}}}{2} \begin{matrix} \swarrow \sqrt{7} \\ \searrow \sqrt{3} \end{matrix}$$

$$= \sqrt{7} - \sqrt{3} \text{ Ans}$$

Type - 11 (max & min)



① find min and max value of $10 + x^2$

$$10 + 0 = 10 \text{ min}$$

$$10 + \infty = 10 \text{ max}$$

② Find min value of $12 + (x-2)^2$

$$(x-2)^2 = 0$$

$$\text{Put } x = 2$$

$$12 + 0$$

$$12 \text{ Ans}$$

③ $5 - (3a - 2b)^2$ will be max, when $a/b = ?$

$$\left[5 - \underbrace{(3a - 2b)^2}_0 \right] \text{ max}$$

$$3a - 2b = 0$$

$$3a = 2b$$

$$\frac{a}{b} = \frac{2}{3} \text{ Ans}$$

④ Find max & min of $5 + x$

$$-\infty \quad +\infty$$

$$5 - \infty = -\infty \text{ min}$$

$$5 - 0 = 5 \text{ max}$$

a, b are +ve

Arithmetic mean	Geometric mean
A.M	G.M
$\frac{a+b}{2}$	$\sqrt{a \cdot b}$

- (5) If $ab = 16$ find min value of $(a+b)$ where a, b are +ve Number

$$a \cdot b = 16$$

$$\left(\frac{a+b}{2}\right)_{\min} =$$

$$\frac{a+b}{2} \geq \sqrt{ab} \quad | \quad 16$$

$$a+b \geq 8 \quad \underline{\text{Ans}}$$

- (6) If $abcd = 81$, find min value of $(a+b+c+d)$

$$\frac{a+b+c+d}{4} \geq \sqrt[4]{abcd} \quad | \quad 81$$

$$a+b+c+d \geq 12 \quad \underline{\text{Ans}}$$

- (7) If $a+b = 10$, find max value of ab , where a, b are +ve variable

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$5 \geq \sqrt{ab}$$

$$25 \geq ab$$

$$= 25 \quad \underline{\text{Ans}}$$

(7)

If $(x-7)(y-10)(z-12) = 1000$
find min value of $(x+y+z)$

$$V: (x-7), (y-10), (z-12)$$

$$\frac{x+y+z-29}{3} \geq \sqrt[3]{(x-7)(y-10)(z-12)}$$

1000

$$x+y+z-29 \geq 30$$

$$x+y+z \geq 59 \text{ Ans}$$

(10)

If $\overset{5}{a} + \overset{5}{b} + \overset{5}{c} = 15$, find max value of abc ,
where a, b, c are +ve variable

$$= 125 \text{ Ans}$$

(11)

If $a+b+c=12$, find max value of $((a+1)(b+2)c)$,
where a, b, c are +ve variable

$$(a+1) + (b+2) + c = 15$$

$$5 \times 5 \times 5$$

$$= 125 \text{ Ans}$$

(12)

If $a+b+c+d=22$ find max value of $((a-2)(b-4)(c+5)(d-1))$

$$(a-2) + (b-4) + (c+5) + (d-1) = 20$$

$$5 \times 5 \times 5 \times 5 = 625 \text{ Ans}$$

(13)

If $3a + 2b + 2c = 14$ find max value of $(a^3 b^2 c^2)$

$$a/a/a/b/b/c/c$$

$$\frac{3a + 2b + 2c}{7} \geq \sqrt[7]{a^3 b^2 c^2}$$

$$2 \geq \sqrt[7]{a^3 b^2 c^2}$$

$$128 \geq a^3 b^2 c^2 \text{ Ans}$$

(14)

If $a + b = 5$ find max value of $(a^3 b^2)$

$$\frac{1}{3} a / \frac{1}{3} a / \frac{1}{3} a / \frac{1}{2} b / \frac{1}{2} b$$

$$\frac{a+b}{5} \geq \sqrt[5]{\frac{a^3 b^2}{108}}$$

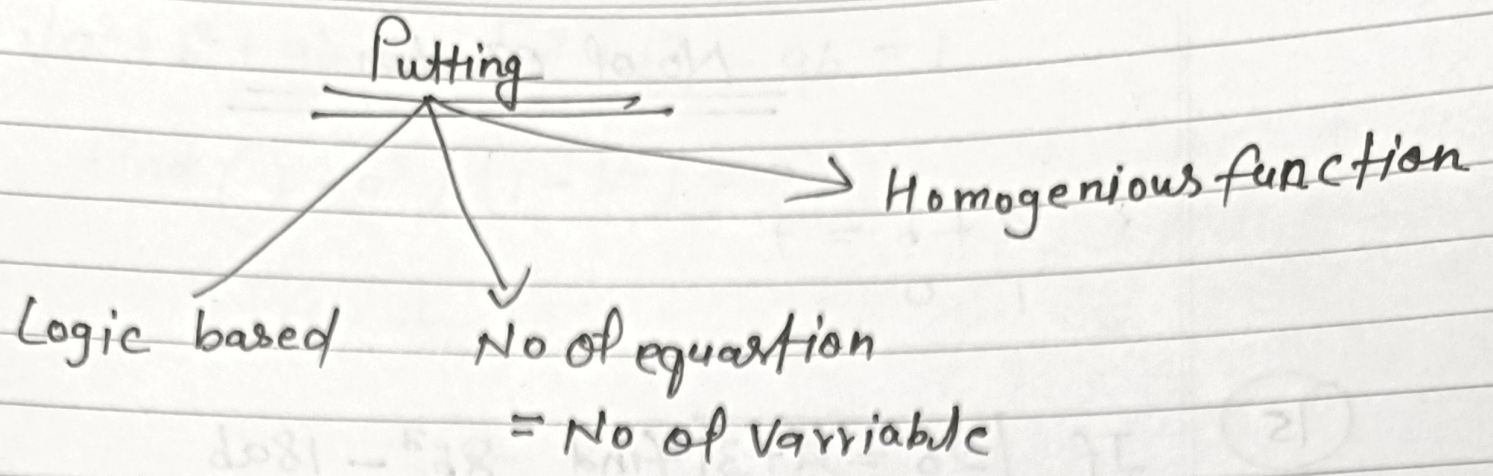
$$1 \geq \sqrt[5]{\frac{a^3 b^2}{108}}$$

$$1 \geq \frac{a^3 b^2}{108}$$

$$108 \geq a^3 b^2 \text{ Ans}$$

(15)

$$\left(x^4 + \frac{1}{x^4} \right)_{\min} = ?$$



Logic based Putting

$$\text{If } a + b + c = 0$$

$$a = 3$$

$$b = -2$$

$$c = -1$$

$$\text{If } a^2b + b^2c + c^2a = 0$$

$$a = 2$$

$$b = -1$$

$$c = 2$$

$$\text{If } \sqrt{a} + \sqrt{b} = \sqrt{c}$$

$$a = 1$$

$$b = 1$$

$$c = 4$$

$$\text{If } a \cdot b \cdot c = 1$$

$$a = b = c = 1$$

No of equation

$$\begin{array}{r} a+b=7 \\ 7 \quad 0 \end{array}$$

(15) If $2b = a+3$ find $8b^3 - 18ab$

$$a=0, \quad b=\frac{3}{2}$$

$$\frac{8 \times 27}{8}$$

27 Ans

(16) If $a+b+\sqrt{ab}=4$ find $\frac{a+b-\sqrt{ab}}{a^2+b^2+\sqrt{ab}}$

$$\begin{array}{l} b=0 \\ a=4 \end{array}$$

$$\begin{array}{l} \frac{a}{a^2} \\ = \frac{1}{a} \end{array}$$

$$= \frac{1}{4}$$

$$= 0.25 \text{ Ans}$$

(17) If $a+b+c=6$ & $ab+bc+ca=11$

find $bc(b+c) + ca(c+a) + ab(a+b) + 3abc$

$$\begin{array}{l} \text{Let} \\ c=0 \end{array}$$

$$\begin{array}{l} 11 \times 6 \\ = 66 \text{ Ans} \end{array}$$

(18) If $p=2q=4$ Find $p^3-8q^3-24pq-64$

$$64-64$$

(5)

$$\text{If } \sqrt{a^2 + b^2 + ab} + \sqrt{a^2 + b^2 - ab} = 1$$

$$\text{find } (1 - a^2)(1 - b^2) = ?$$

$$\left(1 - \frac{1}{4}\right)1$$

$$\frac{3}{4} \text{ Ans}$$

$$a + a = 1$$

$$2a = 1$$

$$a = \frac{1}{2} \text{ Ans}$$

$$\begin{aligned} x + \frac{1}{x} &= 1 & x^3 &= -1 \\ x + \frac{1}{x} &= -1 & x^3 &= 1 \end{aligned}$$

$$\begin{aligned} x + \frac{1}{x} &= \sqrt{3} \\ x^6 &= -1 \end{aligned}$$

(3)

$$\text{If } \frac{L}{m} + \frac{m}{N} + \frac{N}{L} = 0, \text{ then } \frac{NL}{m^2} + \frac{m^2}{LN} =$$

$$N = 0$$

$$m = 0$$

$$\frac{12}{0} = K$$

$$12 = K \times 0$$

K is not defined Ans

$$\frac{12}{6} = K$$

$$12 = K \times 6$$

↓

$$K = 2$$

Solve

$$N=1$$

$$m=1$$

$$L + \frac{1}{L} = -1$$

$$L^3 = 1$$

$$L + \frac{1}{L} = -\frac{1}{L^3}$$

$$-1 - \frac{1}{L}$$

$$-2 \text{ Ans}$$

(6) If $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$ find

$$\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b}$$

$$c=0$$

$$b=1$$

$$a + \frac{1}{a} = 1$$

$$a^3 = -1$$

$$\frac{a^2 + 1}{a}$$

$$\frac{a^3 + 1}{a}$$

$$\frac{-1 + 1}{a} = \frac{0}{a} = 0 \text{ Ans}$$

(7) If $x = \sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - \sqrt{a^2 + b^3}} = 0$
find $x^3 + 3bx$

$$x = \sqrt[3]{2a}$$

$$x^3 = 2a \text{ Ans}$$

(4) If $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$ find value of

$$C + \frac{1}{a} = ?$$

$$C = 0 \times$$

$$b = 2$$

$$C = 1 \times$$

$$a = \frac{1}{2}$$

$$C = -1 \checkmark$$

$$-1 + 2$$

$$1 \text{ Ans}$$

(2)

If $2z = x + y$, find $\frac{x}{x-z} + \frac{y}{y-z}$

$$x = 1 \checkmark$$

$$y = 1 \checkmark$$

$$z = 0$$

$$0 = x + 1$$

$$x = -1$$

$$\frac{1}{1} + \frac{1}{1}$$

$$1 + 1$$

$$2 \text{ Ans}$$

(3)

If $x + y = 36$, find $(x-27)^3 + (y-9)^3$

$$\downarrow \quad \downarrow$$

$$27 + 9$$

$$0 + 0$$

$$= 0 \text{ Ans}$$

(6)

If $abc = 5$, find

$$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+5c^{-1}} + \frac{1}{1+\frac{c}{5}+a^{-1}}$$

$$a = 1$$

$$b = 1$$

$$c = 5$$

$$\frac{1}{3} + \frac{1}{1+1+1}$$

$$\frac{1}{3} + \frac{1}{3}$$

$$1 \text{ Ans}$$

(7) If $a + \frac{3}{b} = 3$, $b + \frac{1}{c} = 1$ find $a(c-1) - abc$

$$b = 2$$

$$c = -1$$

$$a = 3 - \frac{3}{2}$$

$$a = \frac{3}{2}$$

Ans

$$\frac{3}{2}(-2) - \frac{3}{2} \times 2 \times -1$$

$$-3 + 3$$

$$0$$

Ans

(9) If $x^3 + \frac{3}{x} = 4(a^3 + b^3)$ $3x + \frac{1}{x^3} = 4(a^3 - b^3)$

find $\frac{2a^2 - 1}{2b^2 + 1} = ?$

$$ax^2 - 2 + 1 = 0$$

$$2a = 1$$

$$a = \frac{1}{2}$$

$$x = 1$$

$$a = 1$$

$$b = 0$$

$$+b = +2$$

$$b = 2$$

$$= 1 \text{ Ans}$$

(10) If $ab - b + 1 = 0$ and $bc - c + 1 = 0$ find

$$abc + ac - a = ?$$

$$\frac{1}{2} \times 2 \times -1 - \frac{1}{2}(-1) - \frac{1}{2}$$

$$c = 1$$

$$ax^2 - 2 + 1 = 0$$

$$2a = 1$$

$$a = \frac{1}{2}$$

$$+b = +2$$

$$b = 2$$

(11)

If $a+b+c=0$ find

Date: / /

$$\frac{1}{a^2+b-c^2} + \frac{1}{b^2+c^2-a} + \frac{1}{c^2+a^2-b^2}$$

$$\frac{1}{9+4-1} + \frac{1}{4+1-9} + \frac{1}{1+9-4}$$

$$\frac{1}{12} - \frac{1}{14} + \frac{1}{16} = \frac{1-3+2}{12} = \frac{0}{12} = 0 \text{ Ans}$$

(12)

If $ab+bc+ca=0$ find

$$\frac{1}{a^2-bc} + \frac{1}{b^2-ca} + \frac{1}{c^2-ab}$$

$$\frac{1}{4+2} + \frac{1}{1-4} + \frac{1}{4+2}$$

$$\frac{1}{6} - \frac{1}{3} + \frac{1}{6} = \frac{1}{3} - \frac{1}{3} = 0 \text{ Ans}$$

$$a=2$$

$$b=-1$$

$$c=2$$

(3)

If $x = a + \frac{1}{a}$ and $y = a - \frac{1}{a}$ find

$$\sqrt{x^4 + y^4 - 2x^2y^2}$$

$$\sqrt{16}$$

$$x=2$$

$$y=0$$

$$\text{Put } a=1$$

Put
 $a = 1$

$x = 2$
 $y = 0$

(15)

If $2x = a$

Solve $\Rightarrow \frac{1}{a^2 + ax + x^2} - \frac{1}{a^2 - ax + x^2} - \frac{2ax}{a^4 - x^4 + a^2x^2} = ?$

$x = 0$
 $a = 1$

$\frac{1}{a^2} - \frac{1}{a^2}$

0 Ans

(16)

$\frac{1}{a-b} - \frac{1}{a+b} - \frac{2b}{a^2+b^2} - \frac{4b^3}{a^4+b^4} - \frac{8b^7}{a^8-b^8}$

$b = 0$
 $a = 1$

$\frac{1}{a} - \frac{1}{a}$

0 Ans

(17)

If $2x = \sqrt{a} + \frac{1}{\sqrt{a}}$ find $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}$

Solve

Let

$a = 1$

$x = 1$

0 Ans

(18)

If $\sqrt[3]{a} + \sqrt[3]{b} = \sqrt[3]{c}$ find

$(a+b-c)^3 + 27abc$

$a = 0$
 $b = 0$
 $c = 0$

= 0 Ans

(19)

Solve - $\frac{x^2 - 8x + 6}{x^2 - 9x + 20} \div \frac{x^2 - 3x + 2}{x^2 - 5x + 4}$

$x=0$

$\frac{6}{20} \times \frac{4}{2}$
5

$\frac{3}{5}$ Ans

(20)

If $b+c=ax$, $c+a=by$, $a+b=cz$ find

$$\frac{1}{9} \left(\frac{1}{x+1} + \frac{1}{y-1} + \frac{1}{z+1} \right)$$

$$x = \frac{b+c}{a}$$

$$x+1 = \frac{b+c}{a} + 1$$

$$\frac{x+1}{1} = \frac{a+b+c}{a}$$

$$\frac{1}{x+1} = \frac{a}{a+b+c}$$

$$\frac{1}{9} \left(\frac{a}{a+b+c} + \frac{b}{a+b+c} + \frac{c}{a+b+c} \right)$$

$$\frac{1}{9} \left(\frac{a+b+c}{a+b+c} \right) = \frac{1}{9} \underline{\underline{Ans}}$$

Tricks

$$b+c=ax$$

$$c+a=by$$

$$a+b=cz$$

$$a=b=c=1$$

$$x=y=z=2$$

← Logger Putting

$$\frac{1}{9} \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3} \right)$$

$$= \frac{1}{9} \underline{\underline{Ans}}$$

(21) If $x^2 + 4y^2 = 53$ and $x - 2y = 5$ find
 $x^3 - 8y^3 = ?$

$x = 7$
 $y = 1$

$343 - 8$
 $= 335$ Ans

(22) $\frac{a^3(b+c)}{(a-b)(a-c)} + \frac{b^3(c+a)}{(b-c)(b-a)} + \frac{c^3(a+b)}{(c-a)(c-b)}$

$a = 4$
 $b = 1$
 $c = 0$

$\frac{64 \times 1}{3 \times 4} + \frac{1 \times 4}{1 \times -3}$
 $\frac{16}{3} - \frac{4}{3}$

Ans = $\boxed{ab + bc + ca}$ Ans

(23) $\frac{a(b-c)^2}{(c-a)(a-b)} + \frac{b(c-a)^2}{(a-b)(b-c)} + \frac{c(a-b)^2}{(b-c)(c-a)}$

$a = 4$
 $b = 1$
 $c = 0$

$\frac{4 \times 1}{-4 \times 3} + \frac{1 \times 16}{3 \times 1}$

$-\frac{1}{3} + \frac{16}{3}$

5 $\rightarrow$ $\boxed{a+b+c}$

(25)

$$\frac{1}{bc(a+b)(a-c)} + \frac{1}{ca(b-c)(b-a)} + \frac{1}{ab(c-a)(c-b)}$$

$$a=3 \quad \frac{1}{2 \times 1 \times 2} + \frac{1}{3 \times 1 \times -1} + \frac{1}{6 \times -2 \times -1}$$

$$b=2$$

$$c=1$$

$$\frac{1}{4} - \frac{1}{3} + \frac{1}{12}$$

$$\frac{3-4+1}{12} = 0 \quad \underline{\text{Ans}}$$

(27)

If $(a^2+b^2)^3 = (a^3+b^3)^2$ then $\frac{a}{b} + \frac{b}{a}$

~~b=2~~

$$b=1$$

$$(a^2+1)^3 = (a^3+1)^2$$

$$a^6+1+3a^2(a^2+1) = a^6+1+2a^3$$

$$3a^2(a^2+1) = 2a^3$$

$$a + \frac{1}{a} = \frac{2}{3}$$

$$3(a^2+1) = 2a$$

(28)

If $xy(x+y) = 1$ then $\frac{1}{x^3 y^3} - x^3 - y^3$

$$y=1$$

$$x(x+1)$$

$$x+1 = \frac{1}{x}$$

$$x \cdot \frac{1}{x} = -1$$

$$\frac{1}{x^3} - x^3 - 1$$

$$x^3 + 3x - 4 = 1$$

$$= 1-3 = -2$$

$$x^3 - \frac{1}{x^3} - 1$$

$$4-1=3$$

9 If $\sqrt{(1-p^2)(1-q^2)} = \frac{\sqrt{3}}{2}$ find

$$\sqrt{ap^2 + 2q^2 + 2pq} + \sqrt{2p^2 + 2p^2 - 2pq}$$

Solve $q = 0$

$$\sqrt{1-p^2} = \frac{\sqrt{3}}{2}$$

$$1-p^2 = \frac{3}{4}$$

$$p^2 = \frac{1}{4}$$

$$2\sqrt{2}p$$

$$2\sqrt{2} \times \frac{1}{2}$$

$$= \sqrt{2} \text{ Ans}$$

10 If $a^4 + 1 = \left(\frac{a^2}{b^2}\right)(4b^2 - b^4 - 1)$ find $a^4 + b^4$

$$a^4 + 1 = a^2 \times 2$$

$$a^4 - 2a^2 + 1 = 0$$

$$p^2 - 2p + 1 = 0$$

$$(p-1)^2 \cdot (a^2-1)^2 = 0$$

$$a^2 - 1 = 0$$

$$1 + 1 = 2 \text{ Ans}$$

Type - 13

C & D Rule

$$\left(\frac{a}{b} \right) = \left(\frac{c}{d} \right)$$

$$\frac{a \oplus b}{a \ominus b} = \frac{c \oplus d}{c \ominus d}$$

$$\frac{a}{b} + 1 = \frac{c}{d} + 1$$

$$\frac{a+b}{b} = \frac{c+d}{d} \quad \text{--- (1)}$$

$$\frac{a}{b} - 1 = \frac{c}{d} - 1$$

$$\frac{a-b}{b} = \frac{c-d}{d} \quad \text{--- (2)}$$

$$\boxed{\frac{a+b}{a-b} = \frac{c+d}{c-d}}$$

① If $\frac{x+\sqrt{5}}{x-\sqrt{5}} = 2$, find x

$$\frac{(\cancel{x} + \cancel{\sqrt{5}}) + (\cancel{x} - \cancel{\sqrt{5}})}{(\cancel{x} + \cancel{\sqrt{5}}) - (\cancel{x} - \cancel{\sqrt{5}})} = \frac{2+1}{2-1}$$

$$\frac{\cancel{2x}}{\cancel{2}\sqrt{5}} = \frac{3}{1}$$

$$= 3\sqrt{5} \text{ Ans}$$

Trick

$$\frac{x + \sqrt{5}}{x - \sqrt{5}} = \frac{2}{1} \text{ find } x$$

$$\frac{x}{\sqrt{5}} = \frac{3}{1}$$

$$3\sqrt{5} \text{ Ans}$$

2) $\frac{3x+2y}{3x-2y} = \frac{4}{3}$ then find out the value of

$$\frac{(3x+2y) + (3x-2y)}{(3x+2y) - (-3x-2y)} = \frac{4+3}{4-3}$$

$$\frac{2 \times 3x}{2 \times 2y} = \frac{x}{y} = \frac{14}{3} \text{ Ans}$$

3) If $x:y = 2:1$ then $(x^2 - y^2) : (x^2 + y^2)$

$$(4-1) : (4+1)$$

$$3:5 \text{ Ans}$$

4) If $(4p+6q)(4r-6s) = (4p-6q)(4r+6s)$.

then which is correct -

$$\frac{4p+6q}{4p-6q} = \frac{4r+6s}{4r-6s}$$

$$\frac{4p}{62} = \frac{4r}{65}$$

$$= \frac{p}{9} = \frac{r}{5} \quad \underline{\text{Ans}}$$

(5)

If $\frac{\sqrt{x+4} + \sqrt{x-4}}{\sqrt{x+4} - \sqrt{x-4}} = 2$, then find $x = ?$

$$\frac{\sqrt{x+4}}{\sqrt{x-4}} = \frac{3}{1}$$

$$\frac{x+4}{x+4} = \frac{9}{1}$$

$$\frac{x}{4} = \frac{10}{82}$$

$$x = 5 \quad \underline{\text{Ans}}$$

(6)

$a = \frac{\sqrt{x+2} + \sqrt{x-2}}{\sqrt{x+2} - \sqrt{x-2}}$, then find out the value $a^2 - ax$

$$\frac{q+1}{q-1} = \frac{\sqrt{x+2}}{\sqrt{x-2}}$$

$$\frac{q^2+1+2q}{q^2+1-2q}$$

$$\frac{q^2+1}{2q} = \frac{x}{2}$$

$$q^2+1 = qx$$

Trick

Let
 $x = 2$
 $\alpha = 1$

$$1 - 1 \times 2 = -1 \text{ Ans}$$

(7) If $x = \frac{\sqrt{m+3n} + \sqrt{m-3n}}{\sqrt{m+3n} - \sqrt{m-3n}}$ then

$$3nx^2 + 3n = ?$$

$$3 \times 1 \times 1 + 3 \times 1$$

(6) Ans
 $2 \times 3 \times 1$

$$2mx$$

$m = 3$
 $n = 1$
 $x = 1$ Ans

(8) If $x = \frac{\sqrt[3]{m+1} + \sqrt[3]{m-1}}{\sqrt[3]{m+1} - \sqrt[3]{m-1}}$ then find out
 value of $x^3 - 3mx^2 + 3x - m$

$$1 - 3 \times 1 \times 1 + 3 - 1$$

$m = 1$
 $x = 1$ = 0 Ans

(9) If $\frac{x^3 + 3x}{3x^2 + 1} = \frac{189}{61}$, then find out

the value of $x = ?$

$$\frac{(x^3 + 3x) + (3x^2 + 1)}{(x^3 + 3x) - (3x^2 + 1)} = \frac{250}{128}$$

$$(x+1)^3 = \frac{125}{64}$$

$$\frac{x+1}{x-1} = \frac{5}{4}$$

(12)

If $x = \frac{2\sqrt{24}}{\sqrt{8} + \sqrt{3}}$,

then find

$$\frac{x + \sqrt{3}}{x - \sqrt{3}} +$$

$$\frac{x + \sqrt{8}}{x - \sqrt{8}}$$

$$\frac{x}{\sqrt{3}} = \frac{2\sqrt{24} + \sqrt{8} + \sqrt{3}}{\sqrt{8} + \sqrt{3}} \quad \text{--- (1)}$$

$$\frac{3\sqrt{8}}{\sqrt{8} - \sqrt{3}} +$$

$$\frac{x}{\sqrt{8}} = \frac{2\sqrt{3} + \sqrt{3}}{\sqrt{8} + \sqrt{3}} + \frac{3\sqrt{3} + \sqrt{8}}{\sqrt{3} - \sqrt{8}} \quad \text{--- (2)}$$

$$\frac{3\sqrt{8} + \sqrt{3}}{\sqrt{8} - \sqrt{3}} - \frac{(3\sqrt{3} + \sqrt{8})}{\sqrt{8} - \sqrt{3}}$$

$$3\sqrt{8} + \sqrt{3} - 3\sqrt{3} - \sqrt{8} = 2(\sqrt{8} - \sqrt{3}) = 2$$

(11)

If $x = \frac{4\sqrt{15}}{\sqrt{5} + \sqrt{3}}$,

then find out the

value of $\frac{x + \sqrt{12}}{x - \sqrt{12}}$

$$+ \frac{x + \sqrt{20}}{x - \sqrt{20}} = ?$$

$$+ \frac{x + \sqrt{20}}{x - \sqrt{20}} = ?$$

$$= ?$$

Ans

2

(13)

If $x = \frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}}$, then $\frac{x + \sqrt{3}}{x - \sqrt{3}} - \frac{x + \sqrt{2}}{x - \sqrt{2}} = ?$

$$\frac{x}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}} \quad \text{--- (1)}$$

$$\frac{x}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{3} + \sqrt{2}} \quad \text{--- (2)}$$

$$\frac{2\sqrt{2} + \sqrt{3}}{-\sqrt{3}} - \left(\frac{2\sqrt{3} - \sqrt{2}}{-\sqrt{2}} \right)$$

$$- \left(\frac{4 + \sqrt{6} - 6 + \sqrt{6}}{\sqrt{6}} \right)$$

$$\frac{2}{\sqrt{6}} = \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2} \times \sqrt{3}}$$

$$= \frac{\sqrt{2}}{\sqrt{3}} \text{ Ans}$$

(8) If $(a+b) : \sqrt{ab} = 4:1, a > b$, then $a:b = ?$

$$\frac{a+b}{2\sqrt{ab}} = \frac{4}{1 \times 2}$$

$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3}{1}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\frac{(\sqrt{a} + \sqrt{b})^2}{(\sqrt{a} - \sqrt{b})^2}$$

$$\frac{a}{b} = \frac{4 + 2\sqrt{3}}{4 - 2\sqrt{3}}$$

Type-1BSpecial mixed question

If $x + \frac{1}{x} = k$

① $x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$

② $x^7 + \frac{1}{x^7} = \left(x^4 + \frac{1}{x^4}\right)\left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$

$x - \frac{1}{x} = k$

① $x^5 - \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 - \frac{1}{x^2}\right) + \left(x - \frac{1}{x}\right)$

② $x^7 - \frac{1}{x^7} = \left(x^4 + \frac{1}{x^4}\right)\left(x^3 - \frac{1}{x^3}\right) + \left(x - \frac{1}{x}\right)$

① If $x > 0$, $x^4 + \frac{1}{x^4} = 254$, find $x^5 + \frac{1}{x^5}$

$x - \frac{1}{x} = \sqrt{18}$

$$\left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$$

$$5\sqrt{18} \times 16 - \sqrt{18}$$

$$\sqrt{18}(240 - 1)$$

$$3\sqrt{2} \times 239$$

$$717\sqrt{2} \text{ Ans}$$

(2)

If $x > 0$, $x^4 + \frac{1}{x^4} = 254$, find $x^7 + \frac{1}{x^7}$

Solve

$$x + \frac{1}{x} = \sqrt{18}$$

$$x^2 + \frac{1}{x^2} = 16$$

$$x^4 + \frac{1}{x^4} = 254$$

$$\left(x^4 + \frac{1}{x^4}\right) \left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$$

$$254 \times 16\sqrt{18} - \sqrt{18}$$

$$3\sqrt{2} \times (3809)$$

$$11427\sqrt{2} \text{ Ans}$$

(3)

$x - \frac{1}{x} = -9$, find $x^5 - \frac{1}{x^5}$

Solve

$$x + \frac{1}{x} = \sqrt{85}$$

$$\left(x^3 + \frac{1}{x^3}\right) \left(x^2 - \frac{1}{x^2}\right) + \left(x - \frac{1}{x}\right)$$

$$82\sqrt{85}x - 9\sqrt{85} - 9$$

$$-9\sqrt{85} \times 82 + 1$$

$$-9(6971)$$

$$= 62739 \text{ Ans}$$

(4)

$$\left(k - \frac{1}{k}\right) \left(k^2 + \frac{1}{k^2}\right) \left(k^4 + \frac{1}{k^4}\right) \left(k^8 + \frac{1}{k^8}\right) \left(k^{16} + \frac{1}{k^{16}}\right)$$

Solve

$$\frac{\left(k + \frac{1}{k}\right)}{\left(k + \frac{1}{k}\right)}$$

$$= \left(k^{32} - \frac{1}{32}\right)$$

$$\frac{1}{k + \frac{1}{k}}$$

Ans

(5) $(1+x)(1-x)(1+x^2)(1+x^4)(1+x^8) = p$

$$1-x^2$$

$$(1-x^4)$$

$$(1-x^8)$$

$$1-x^{16} \text{ Ans}$$

(6) If $\frac{1}{x+1} + \frac{2}{y+2} + \frac{1011}{z+1011} = 1$, find

$\frac{x}{x+1} + \frac{y}{y+2} + \frac{2}{z+1011} = p$ (2)

Solve $\frac{1}{x+1} + \frac{2}{y+2} + \frac{1011}{z+1011} = 1$

$p = 2$ Ans

(7) If $\frac{a}{1-2a} + \frac{b}{1-2b} + \frac{c}{1-2c} = 1$ find

$\frac{1}{1-2a} + \frac{1}{1-2b} + \frac{1}{1-2c} = p$ (2)

2 $\frac{1}{1-2a} - \frac{2a}{1-2a}$

$1+1+1 = p-2$

8

If $\frac{a}{x-1} + \frac{4b}{y-2b} + \frac{9c}{z-3c} = 6 - a$, Solve

$$\frac{ax}{x-1} + \frac{2y}{y-2b} + \frac{3z}{z-3c} = (P)$$

$$\frac{ax}{x-1} - \frac{a}{x-1} = P - (6a)$$

$$a + 5 = P - 6 + 0$$

$$P = 11 \text{ Ans}$$

9

"Addition or Subtraction of two equation"

9

If $x = 78$, find

$$x^7 - 79x^6 + 79x^5 - 79x^4 + 79x^3 - 79x^2 + 79x - 1$$

$$\cancel{x^7 - 78x^6 - x^6 + 78x^5 + x^5 - 78x^4 + 78x^3 + x^3 - 78x^2 + 78x + 1}$$

$$= \text{Ans} - 1$$

If $x = 7$ find

$$x^6 - 8x^5 + 8x^4 - 8x^3 + 10x^2 - 8x + 13$$

$$\cancel{x^6 - 7x^5 + 7x^4 + x^4 - 7x^3 - 1x^3 + 7x^2 + 3x^2 - 7x + 13}$$

$$x + 13 = 49 + 6 = 104$$

⑪ If ~~4a~~ $4a = 3b = 36^c = \text{find } c$

$$4a = 3b = 36^c = K$$

$$4 = K^{\frac{1}{a}} \quad \text{--- (1)}$$

$$3 = K^{\frac{1}{b}} \quad \text{--- (2)}$$

$$36 = K^{\frac{1}{c}} \quad \text{--- (3)}$$

$$K^{\frac{2}{5}} \times K^{\frac{1}{a}} = K^{\frac{1}{c}}$$

$$\frac{2}{5} + \frac{1}{a} = \frac{1}{c}$$

$$\frac{1}{c} = \frac{2a+b}{ab}$$

$$= \frac{ab}{2a+b} \quad \underline{\underline{\text{Ans}}}$$

⑫ If $x^{2a} = y^{2b} = z^{2c} \neq 0$ & $x^2 = y^2$,

find $\frac{ab+bc+ca}{bc}$

$$x = K^{\frac{1}{2a}}$$

$$K^{\frac{1}{a}} = K^{\frac{1}{2b} + \frac{1}{2c}}$$

$$2bc = ab + ac$$

$$3bc = ab + ac + bc$$

$$+ bc$$

$$\frac{1}{a} = \frac{1}{2b} + \frac{1}{2c}$$

$$\frac{1}{a} = \frac{cb+ab}{2bc}$$

$$\underline{\underline{3}} \quad \underline{\underline{\text{Ans}}}$$

(13)

If $3^a = 27^b = 81^c$ and $abc = 144$ find

$$12 \left(\frac{1}{a} + \frac{1}{2b} + \frac{1}{5c} \right)$$

Solve

$$3^{\textcircled{a}} = 3^{\textcircled{3b}} = 3^{\textcircled{4c}}$$

$$1a = 3b = 4c = 12$$

$$a : b : c$$

$$12K : 4K : 3K$$

$$144K^3 = 144$$

$$12 \left(\frac{1}{12} + \frac{1}{8} + \frac{1}{16} \right)$$

$$\cancel{12} \left(\frac{10+15+8}{\cancel{120}} \right) = \frac{33}{10}$$

(14)

If $x = 3 + 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$ find $x^3 - 9x^2 + 18x$

$$a = 3$$

$$b = 3^{\frac{1}{3}}$$

$$c = 3^{\frac{2}{3}}$$

$$a^3 + b^3 + c^3 - 3abc$$

$$\cancel{27} + 3 + 9 - 3 \times \cancel{3} \times 3^2$$

$$= 12 \text{ Ans}$$

(15)

If $x = 5 - 5^{\frac{2}{3}} + 5^{\frac{1}{3}}$ find

$$\textcircled{3} x - 15\textcircled{2} + 60\textcircled{1}$$

$$a^3 + b^3 + c^3 - 3abc$$

$$a = 5^1$$

$$b = 5^{\frac{1}{3}}$$

$$125 - 25 + -3(-2)$$

$$105 + 75$$

$$= 180 \text{ Ans}$$

(16)

Concept (+1)

$$\text{If } xy + x + y = 23$$

$$xz + z + x = 41$$

$$yz + y + z = 27, \text{ find } z$$

$$1 + xy + x + y = 24$$

$$(1+x) + y(1+x) = 24 = 4 \times 6$$

$$(1+x)(1+y) = 24 = 4 \times 6$$

$$(1+x)(1+z) = 42 = 6 \times 7$$

$$(1+y)(1+z) = 28 = 7 \times 4$$

$$1+z = 7$$

$$z = 6 \text{ Ans}$$

(17)

A & B are + integer, $A+B+AB = 186$

Find positive difference between

A & B given $A, B \leq 20$

$$1 + A + B + AB = 186 + 1$$

$$(1+A)(1+B) = 187$$

$$(1+A)(1+B) = 187$$

$$17 \times 11$$

$$A = 16$$

$$B = 10$$

$$\text{Ans} \rightarrow 6$$

(18)

If $xyz + xy + yz + zx + x + y + z = 384$,

given x, y, z are positive integers
 find $x + y + z$

$$(1+x)(1+y)(1+z)$$

$$(1+y+x+xy)(1+z)$$

$$1+y+x+xy+z+zyz$$

$$1+$$

$$(1+x)(1+y)(1+z)$$

$$= 385$$

$$5 \times 7 \times 11$$

$$x = 4$$

$$y = 6$$

$$z = 10$$

$$= 20 \text{ Ans}$$

Q- $(x+y+1)[xy+1(x+y)] - 7xy = 0$
 $x+y=0$

$$x \neq -7$$

$$y \neq -7$$

If $x + \frac{1}{x} = 2$ $x = 1$

If $x + \frac{1}{x} = -2$, $x = -1$

$$\frac{\sqrt{5}}{(x+y+\sqrt{5})^3} \cdot \frac{1}{5}?$$

$$\frac{\sqrt{5}}{5\sqrt{5}} - \frac{1}{5} = 0 \text{ Ans}$$

Q22 If $x + \frac{1}{x} = 2$, then find out the value of $x^{19} + \frac{1}{x^{19}}$

$$x + x = 1$$

$$1+1$$

$$= 2 \text{ Ans}$$

Q23 If $x + \frac{1}{x-3} = 5$, then find out the value of $(x-3)^3 + \frac{1}{(x-3)^3}$

$$(x-3) + \frac{1}{(x-3)} = 2$$

$$x-3 = 1$$

$$1+1 = 2 \text{ Ans}$$

(24) If $a+b=10$ and $\sqrt{\frac{b}{a}} - 13 = -\sqrt{\frac{a}{b}} - 11$,
then what is the value of

$$4a^2 + 6b^2 + 3ab = ?$$

Solve

$$\sqrt{\frac{b}{a}} + \sqrt{\frac{a}{b}} = 2$$

$$a = b$$

$$100 + 150 + 75$$

$$= 325 \text{ Ans}$$

(25) If $\left(\frac{a^2}{b^3}\right) + \left(\frac{b^3}{a^2}\right) = -2$ then find out the
value of $(a^6 + b^9 + 3a^2b^3)(a^2 + b^3) = ?$

$$\frac{a^2}{b^3} = -1$$

$$a^2 = -b^3$$

$$\rightarrow (a^2 + b^3)^3$$

$$= 0 \text{ Ans}$$

(6) If $(4x-7)(9-4x) = 1$, find

$$(x-3)^3 + \frac{1}{(x-3)^3}$$

$$4x-7 = \frac{1}{9-4x}$$

$$4x-9 = \frac{1}{4x-9} - 2$$

$$4x-9 = \frac{1}{4x-9} \quad (4x-9) + \frac{1}{4x-9} =$$

$$4x = 8$$

$$x = 2$$

$$\begin{array}{r} -1 -1 \\ -2 \text{ Ans} \end{array}$$

Concept

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k \quad \text{Let}$$

$$x = ak \quad \text{--- (1)}$$

$$y = bk \quad \text{--- (2)}$$

$$z = ck \quad \text{--- (3)}$$

$$k = \frac{x+y+z}{a+b+c}$$

$$(1) - (2) + (3)$$

$$k = \frac{x-y+z}{a-b+c}$$

$$(1) - (2) - (3)$$

$$k = \frac{x-y-z}{a-b-c}$$

Note $\Rightarrow$

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(29)

If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, then which Correct

$$\frac{x+y+z}{a+b+c} \quad \underline{\underline{\text{Ans}}}$$

(27)

If $\frac{x^2}{by+Cz} = \frac{y^2}{ax+Cz} = \frac{z^2}{ax+by} = (2)$ then

$$\frac{a}{x+2a} + \frac{b}{y+2b} + \frac{c}{z+2c} = ?$$

Putting

$$x=y=z=4$$

$$a=b=c=1$$

$$1 \frac{1}{6} \times 3$$

$$= \frac{1}{2} \quad \underline{\underline{\text{Ans}}}$$

(31)

If $\frac{a+b}{am+bn} = \frac{b+c}{mb+cn} = \frac{c+a}{mc+na}$, then

which is Correct.

$$\frac{2(a+b+c)}{m(a+b+c)+n(a+b+c)}$$

$$\frac{2}{m+n} \quad \underline{\underline{\text{Ans}}}$$

(32)

If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, then $\frac{ax - by}{(a+b)(x-y)} +$

$\frac{by - cz}{(b+c)(y-z)} + \frac{cz - ax}{(c+a)(z-x)} = ?$

$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$

$x = ak$

$y = bk$

$z = ck$

3 Ans

$\frac{a^2k - b^2k}{(a+b)(ax - by)} = 1$

(30)

If $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$, then which is correct.

Solve

$\frac{x}{b+c} = k$

$x = k(b+c) \quad \text{--- (1)}$

$y = k(c+a) \quad \text{--- (2)}$

$z = k(a+b) \quad \text{--- (3)}$

$\textcircled{1-2}$

$k = \frac{x-y}{b-a}$

$=$

$\frac{x-y}{b-a}$

$= \frac{y-z}{c-b}$

$= \frac{z-x}{a-c}$

Ans

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②

If $x^2 - 2x + 4 = 0$, then $x^5 - 2x^4 + 30$

$$x^3(x^2 - 2x) + 30$$

$$= 4x^3 + 30$$

$$= 4x - 8 + 30$$

Ans

$$(x+2)(x^2 - 2x + 4) = 0$$

$$x^3 + 2 = 0$$

$$x^3 = -8$$

③

If $(x+5)(x+6) + \frac{9}{x(x-1)} = 0$ find

$$2x^2 + 10x + 17$$

$$x(x-1)(x+5)(x+6) + 9 = 0$$

$$(x^2 + 5x)(x^2 + 6x - x - 6) + 9 = 0$$

$$p(p-6) + 9 = 0$$

$$p^2 - 6p + 9 = 0$$

$$(p-3)^2 = 0$$

$$p-3 = 0$$

$$p = 3$$

$$2(x^2 + 5x) + 17$$

$$= 23$$

$$(a+b+c) - \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = m+n+p$$

$$mnp + m+n+p \quad \text{Ans}$$

5) If $xyz = 1$, $\left(x + \frac{1}{z} = 5\right)$, $\left(y + \frac{1}{x} = 29\right)$
find $\left(x + \frac{1}{y}\right) = P$

Solve

$$\left(x + \frac{1}{z}\right) \left(y + \frac{1}{x}\right) \left(z + \frac{1}{y}\right) = 5 \times 29 P$$

$$\left(\cancel{xyz} - \frac{1}{\cancel{xyz}}\right) - (x-y-z) + \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 145P$$

$$\left(xy + 1 + \frac{y}{z} + \frac{1}{xz}\right) \left(z + \frac{1}{y}\right) = 145P$$

$$1 + \left(\frac{z}{z}\right) + y + \frac{1}{x} + x + \frac{1}{y} + 1 = 145P$$

$$2 + 34 + P = 145P$$

$$144P = 36$$

$$P = \frac{36}{144} = \frac{1}{4} \quad \text{Ans}$$

If $a^2 - 4a - 1 = 0$, find $a^6 - 76a^3 + 3$