(Quadratic Equation & Cubic Equation) Solution

1. Sol.

one root $= 11 - \sqrt{7}$

second root $= 11 + \sqrt{7}$

sum of roots $= 22$

product of roots $= 114$

Now $x^2 - 22x + 114 = 0$ Ans

2. Sol.

$D = 0$

$b^2 - 4ac = 0$

$c = \frac{b^2}{4a}$

3. Sol.

$\alpha + \beta = -\frac{b}{a} = \frac{-(-1)}{1} = 1$

$\alpha \cdot \beta = \frac{c}{a} = 1$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$\alpha^3 + \beta^3 = -2$

Now $x^2 + 2x + 1 = 0$ Ans

4. Sol.

$\alpha + \beta = -\frac{b}{a} = \frac{-(-1)}{1} = 1$

$\alpha \cdot \beta = \frac{c}{a} = 1$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\alpha^2 + \beta^2 = -1$

$\alpha^2\beta^2 = 1$

Now $x^2 + x + 1 = 0$ Any

5. Sol.

$\alpha + \beta = -\frac{b}{a} = \frac{-(-1)}{1} = 1.$

$\alpha \cdot \beta = \frac{c}{a} = \frac{-1}{1} = -1.$

$\beta = -\frac{1}{\alpha}$

$\alpha - \frac{1}{\alpha} = 1$

$\alpha^2 + \frac{1}{\alpha^2} = 3$

$\alpha^4 + \frac{1}{\alpha^4} = 7$

$\alpha^2 + \frac{1}{\alpha^8} = 47$

6. Sol.

$\alpha + \beta = \frac{-b}{a} = -\frac{(-1)}{1} = 1$

$\alpha \cdot \beta = \frac{c}{a} = \frac{1}{1} = 1$

$\beta + \frac{1}{\alpha} = 1$

$\alpha^2 + \frac{1}{\alpha^2} = -1$

so $\alpha^4 + \frac{1}{\alpha^4} = -1$

$\alpha^4 \cdot \beta^4 = 1,$

$x^2 + x + 1 = 0$ Dy

7. Sol.

we know roots are equal

$D = 0$

$b^2 - 4ac = 0$

$4n^2c^2 - 4 \times (1 + n^2)(c^2 - a^2) = 0$

$c^2 = a^2(1 + n^2)$

8. Sol.

let $ax^2 + bx + c = 0$

If $a + b + c = 0$

Then one root will 1 and another will $\frac{c}{a}$

- 1, $\frac{a+b-c}{a+b+c}$

9. Sol.

$\frac{1}{x+a+b} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\frac{x-(x+a+b)}{x(x+a+b)} = \frac{a+b}{ab}$

$\frac{-1}{x(x+a+b)} = \frac{1}{ab}$

$x^2 + ax + xb + ab = 0$

$x^2 + x(a + b) + ab = 0$

$\alpha + \beta = -(a + b)$

$\alpha \cdot \beta = ab$

$\alpha^2 + \beta^2 = a^2 + b^2$

$\alpha^2\beta^2 = a^2b^2$

Now

$x^2 - (a^2 + b^2)x + a^2b^2$

10. Sol.

$\alpha + \beta = -\frac{b}{a} = \frac{13}{3}$

$\alpha \cdot \beta = \frac{c}{a} = \frac{14}{3}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\left(\frac{13}{3}\right)^2 - 2 \times \frac{14}{3}$

$\alpha^2+\beta^2=\frac{85}{9}$

Now by question:-

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{85}{42}$

11. **Sol.**

$ax^2+bx+c=0$ (roots $\alpha\&\beta$)

Now $cx^2+bx+a=0$ $\left(\text{roots}\frac{1}{\alpha}\&\frac{1}{\beta}\right)$

So $cx^2+bx+a=0$ Ans

12. **Sol.**

$\alpha^5-\frac{1}{\alpha^5}=\left(\alpha^3+\frac{1}{\alpha^3}\right)\left(\alpha^2-\frac{1}{\alpha^2}\right)+\left(\alpha+\frac{1}{\alpha}\right)$ (1)

$\alpha+\beta=\frac{-b}{a}=-\frac{1}{1}$

$\alpha\cdot\beta=\frac{c}{a}=-\frac{1}{1}=-1$ $\left(\text{so }\beta=-\frac{1}{\alpha}\right)$

$\alpha-\frac{1}{\alpha}=-1,\alpha+\frac{1}{\alpha}=\sqrt{5}\ \&\alpha^2-\frac{1}{\alpha^2}=-\sqrt{5}$

$\alpha^3+\frac{1}{\alpha^3}=2\sqrt{5}$

by equation (1) $\alpha^5-\frac{1}{25}=-11\ x^2+11x-1=0$

13. **Sol.**

$\alpha^5+\beta^5=(\alpha^3+\beta^3)(\alpha^2+\beta^2)-\alpha^2\beta^2(\alpha+\beta)$

$\alpha+\beta=-\frac{(-2)}{1}=2$

$\alpha\cdot\beta=\frac{4}{1}=4$

By question

$-\frac{\alpha^5+\beta^5}{\alpha^2\beta^2}=2$

Now $x^2-2x+4=0$ Ans

14. **Sol.**

$\alpha+\beta=8$

$\alpha-\beta=2\sqrt{5}$

Now $\alpha=4+\sqrt{5}$

$\beta=4-\sqrt{5}$

$\alpha^4+\beta^4=\left((4+\sqrt{5})^2\right)^2+\left((4-\sqrt{5})^2\right)^2$

$\alpha^4+\beta^4=1522$

$x^2-1522x+14641=0$

Now

15. **Sol.**

$\alpha-\beta=4$ (given)

$\alpha+\beta=-\frac{b}{a}=\frac{B}{A}$

after solving (1) & (2)

$\alpha=\frac{4A+B}{2A}\&\beta=\frac{B-4A}{2A}$

we know

$\alpha\cdot\beta=\frac{C}{A}$

$\left(\frac{4A+B}{2A}\right)\left(\frac{B-4A}{2A}\right)=\frac{C}{A}$

$B^2-16A^2=4AC$

16. **Sol.**

Sum $A+B\ =\left(-\frac{b}{a}\right)=\frac{A^2}{A}=A$

$B\ =0$

product $A\cdot B\ =\left(\frac{c}{a}\right)=\frac{AB}{A}=B$

A = 1

17. **Sol.**

See solution No (8)

$\alpha=1$

$\beta=\frac{c}{a}=\frac{c(a-b)}{a(b-c)}$

roots are equal

so $\frac{c(a-b)}{a(b-c)}=1$

After solving $\frac{2}{b}=\frac{1}{c}+\frac{1}{a}$

18. **Sol.**

$x^2+24x+119=0$

Sign change: $+17\ \ +7$ → $+17\ \ +7$

$-17,-7$

19. **Sol.**

$6x^2+28x+16=0$

96

24 4

$\frac{-24}{6}$ $\frac{-4}{6}$

$-4,\ \frac{-2}{3}$

20. **Sol.**

$(x-\beta)$

$(x-\alpha)+(x-\beta)$

$2x-(\alpha+\beta)$

We know sum of roots $\alpha+\beta=-\frac{b}{a}=\frac{13}{1}$

$2x-13$

21. **Sol.**

$3x$

22. **Sol.**

$3x-11$

23. Sol.

Sum of the roots $(ck+k)=\frac{-b}{a}$

$k=\frac{-b}{a(c+1)}$

Product of the roots k.ck $=\frac{c}{a}$

$c\cdot\left(\frac{-b}{a(c+1)}\right)^2=\frac{c}{a}$

$\therefore b^2=a\cdot(c+1)^2$

24. Sol.

$2x^2+5x+1=0$

On dividing by $2x$,

$x+\frac{5}{2}+\frac{1}{2x}=0\Rightarrow x+\frac{1}{2x}=-\frac{5}{2}$

$x-\frac{1}{2x}=\sqrt{\left(x+\frac{1}{2x}\right)^2-4\times\frac{1}{2}}$

$x-\frac{1}{2x}=\sqrt{\left(-\frac{5}{2}\right)^2-2}$

$x-\frac{1}{2x}=\sqrt{\frac{25}{4}-2}=\frac{\sqrt{17}}{2}$

25. Sol.

Let $\Rightarrow 3\alpha, 2\alpha$ are roots,

$6\alpha^2=\frac{5}{12}$

$\alpha=\sqrt{\frac{5}{72}}=\pm\frac{1}{6}\sqrt{\frac{5}{2}}$

$5\alpha=\frac{-m}{12}$

$=5+\frac{1}{6}\sqrt{\frac{5}{2}}=\frac{-m}{12}$

$m=\pm5\sqrt{10}$

26. Sol.

$k(21x^2+24)+rx+(14x^2-9)=0$ ... (i)

$k(7x^2+8)+px+(2x^2-3)=0$ ... (ii)

Multiply equation (ii) by 3 and subtract from equation (i),

$k(21x^2+24)+rx+(14x^2-9)-3\times[k(7x^2+8)+px+(2x^2-3)]=0$

$8x^2-3px+rx=0$ ...(iii)

So let roots of equation (iii) are α and β

$\alpha+\beta=\frac{3p}{8}$

$\alpha\beta=\frac{r}{8}$

Equations have both roots common.

so $\frac{3p}{8}=\frac{r}{8}\Rightarrow\frac{p}{r}=\frac{1}{3}$

27. Sol.

Since, α,β are the roots of $6x^2+13x+7=0$. then ,

$\alpha+\beta=\frac{-13}{6}$ and $\alpha\beta=\frac{7}{6}$

$\alpha^2+\beta^2=\left(\frac{-13}{6}\right)^2-2\times\frac{7}{6}=\frac{169}{36}-\frac{7}{3}=\frac{85}{36}$

So, the equation whose roots are α^2,β^2 is given by :

$x^2-\left(\frac{85}{36}\right)x+\frac{49}{36}=0$

$36x^2-85x+49=0$

28. Sol.

$x^2+\alpha\cdot x+\beta=0$

Sum of the roots, $\alpha+\beta=-\alpha$

$\beta+2\alpha=0$

Product of the roots $\alpha\cdot\beta=\beta\Rightarrow\beta(1-\alpha)=0$

As $\beta\neq0\therefore\alpha=1$

and $\beta+2=0\Rightarrow\beta=-2$.

Now, $\alpha-\beta=1-(-2)=3$.

29. Sol.

I method

$PX^2-QX+R=0$,

Let $a=b=1$, then equation $\Rightarrow x^2-2x+1=0$

$P=1, Q=2, R=1$,

$\frac{1}{a^2}+\frac{1}{b^2}+\frac{a}{b}+\frac{b}{a}=4.$

Put the value in option, then (B) satisfied

II method

$a+b=\frac{Q}{P}$

$ab=\frac{R}{P}$,

$\frac{1}{a^2}+\frac{1}{b^2}+\frac{a}{b}+\frac{b}{a}=\frac{a^2+b^2+a^3b+ab^3}{a^2b^2}$

$=\frac{(a^2+b^2)(1+ab)}{a^2b^2}$

$=\frac{[(a+b)^2-2ab]\times(1+ab)}{a^2b^2}$

30. Sol.

For real roots D > 0

$b^2-4ac>0$

$b^2 > 4ac$

31. **Sol.**
Common root = α

$\alpha^2 + a\alpha + 8 = 0$
$\alpha^2 + b\alpha - 8 = 0$
$(a-b)\alpha + 16 = 0 \rightarrow a - b = -16/\alpha$
$2\alpha^2 + (a+b)\alpha = 0 \rightarrow a + b = 2\alpha$

$a^2 - b^2 = 32$

32. **Sol.**
$12x^3 - 998x + 3572 = 0$

$p + q + r = 0, (p+q)^3 + (q+r)^3 + (r+p)^3$

$-r^3 + (-p)^3 + (-q)^3$

$\rightarrow -(p^3 + q^3 + r^3)$

$\rightarrow -3pqr$

$-\frac{3\times(3572)}{4\times3} = 893$

33. **Sol.**
For equal roots $D = 0$

$b^2 = 4ac$

$\Rightarrow (2mc)^2 = 4(1+m^2)(c^2 - a^2)$

$\Rightarrow 4m^2c^2 = 4(c^2 - a^2 + m^2c^2 - m^2a^2)$

$\Rightarrow c^2 = a^2(1+m^2)$

34. **Sol.**
$3x^2 - 13x + 14 = 0$

$\alpha + \beta = 13/3$

$\alpha\beta = 14/3$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$\frac{\frac{169}{9} - \frac{28}{3}}{\frac{14}{3}} = \frac{85}{9} \times \frac{3}{14} = \frac{85}{42}$

35. **Sol.**
$x^2 - 14x + 1 = 0$

$x = \frac{14 \pm \sqrt{(14)^2 - 4(1)(1)}}{2}$

$x = 7 \pm 4\sqrt{3}$

$\alpha = 7 + 4\sqrt{3}$ And $\beta = 7 - 4\sqrt{3}$

$\Rightarrow \alpha = (2+\sqrt{3})^2 \Rightarrow \beta = (2-\sqrt{3})^2$

$\Rightarrow \sqrt{\alpha} = 2 + \sqrt{3} \Rightarrow \sqrt{\beta} = 2 - \sqrt{3}$

$\sqrt{\alpha} + \sqrt{\beta} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

$(\sqrt{\alpha})(\sqrt{\beta}) = (2+\sqrt{3})(2-\sqrt{3}) = 1$

So equation of quadratic equation with roots $\sqrt{\alpha}$ and $\sqrt{\beta}$ is

$x^2 - (\sqrt{\alpha} + \sqrt{\beta})x + \sqrt{\alpha}\sqrt{\beta} = 0$

$x^2 - 4x + 1 = 0$

36. **Sol.**
To find common roots put $x = \text{t}$ and equate both equations,

$\Rightarrow \text{t}^3 - 5\text{t}^2 + 3\text{t} - 9 = \text{t}^3 - 6\text{t}^2 + 8\text{t}$

-15

$\Rightarrow t^2 - 5t + 6 = 0$

$\Rightarrow (t-2)(t-3) = 0$

$\therefore t = 2,3$

Now, on putting these values in both equations,

first eq. $\Rightarrow x^3 - 5x + 3x - 9 = 0$

on putting $x = 2$

$\Rightarrow (2)^3 - 5(2)^2 + 3(2) - 9$

$\Rightarrow 8 - 20 + 6 - 9$

$\Rightarrow -15 \neq 0$

on putting $x = 3$

$\Rightarrow (3)^3 - 5(3)^2 + 3(3) - 9$

$\Rightarrow 9 - 45 + 9 - 9$

$\Rightarrow -36 \neq 0$

Since, these are not equal to zero hence, no roots are common.

37. **Sol.**
We know that two equation

$a_1x^2 + b_1x + c_1 = 0$

$a_2x^2 + b_2 + c_2 = 0$

have common root when

$(c_1a_2 - a_1c_2)^2 = (b_1c_2 - c_1b_2)(a_1b_2 - b_1a_2)$

So, for $x^2 + 5x + 6 = 0$ and $x^2 + kx + 1 = 0$

we have $(5)^2 = (5 - 6x)(x - 5)$

$\Rightarrow 25 = -6x^2 + 35x - 25$

$\Rightarrow 6x^2 - 35x + 50 = 0$

$\Rightarrow x = \frac{5}{2}$ or $\frac{10}{3}$

38. Sol.

p and q are roots of

$x^2 + px + q = 0$

$\Rightarrow p + q = -p$ and $pq = q$

$\Rightarrow pq - q = 0$

$\Rightarrow q(p-1) = 0 \Rightarrow (p-1) = 0 \Rightarrow p = 1$

Adding p in equation $p + q = -p$

$\Rightarrow 1 + q = -1$

$\Rightarrow q = -2$

39. Sol.

We know the roots of an equation

$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

here, $a = 1, b = -4$ and $c = a$

According to question

$\frac{-(-4)+\sqrt{(-4)^2-4a(1)}}{2(1)}$ ………(1)

$= \frac{-(-4)-\sqrt{(-4)^2-4a(1)}}{2(1)}$ ………(2)

Equating eq (1) and (2), we get

$2\sqrt{16 - 4a} = 0$

$16 = 4a$ so, $a = 4$ ans.

Alternate Method :

For real and equal roots,

$D = b^2 - 4ac = 0$

$a = 1, b = -4$ and $c = a$

$\Rightarrow 4^2 - 4a = 0$

$\Rightarrow 4a = 16 \Rightarrow a = 4$